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Spontaneous Emission in Cavities: How Much More Classical Can You Get?[1]

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Cavity-induced changes in atomic spontaneous emission rates are often interpreted in terms of quantum electrodynamical zero-point field fluctuations. A completely classical method of computing this effect in terms of the unquantized normal mode structure of the cavity is presented here. Upon applying the result to a classical dipole radiating between parallel mirrors, we obtain the same cavity correction as that for atomic spontaneous emission in such a cavity. The theory is then compared with a recent experiment in the radio-frequency domain.

1. INTRODUCTION

In 1946, Purcell gave the first indication that the confinement of an excited atom in a cavity should alter the spontaneous emission rate from the free space value.[1] The essence of his argument is as follows: Let the atom–field system be in an initial excited state $|i, 0_{\mathbf{k}}\rangle$, indicating zero photons present. The final state we shall denote by $|f, 1_{\mathbf{k}}\rangle$ showing that a single photon of wave number $\mathbf{k}$ has been emitted. Fermi's golden rule for the transition rate w_{fi} yields

$$w_{\mathrm{fi}} = \frac{2\pi}{\hbar} \rho(\hbar\omega_{\mathbf{k}}) \, |\langle f, 1_{\mathbf{k}}| \, H_{\mathrm{int}} \, |i, 0_{\mathbf{k}}\rangle|^2 \tag{1}$$

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[1] It is a pleasure and an honor to dedicate this paper to Professor Asim O. Barut, who, as my teacher, advisor, and friend, has been a great inspiration to me as well as many, many others.

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where $\hbar$ is Dirac's constant, $H_{\rm int} = \boldsymbol{\mu} \cdot \mathbf{E}$ is the interaction Hamiltonian operator that couples the atom to the electromagnetic field, and ρ is the density of available final photon states for the emitted photon of frequency $\omega_{\mathbf{k}} = \omega_{\rm fi}$, the transition frequency. This density depends on number and structure of the electromagnetic normal modes of the field. Since the modal density depends explicitly on the geometry of the surrounding space, the presence of a conducting cavity, say, would alter the mode density ρ and hence the transition rate $w_{\rm fi}$, Eq. (1). For an atom in a vacuum, one often hears ascribed the notion of a "trigger" for the decay event to the density of electromagnetic vacuum fluctuations at the transition frequency $\omega_{\rm fi}$. With this sort of argument, then, ρ represents the density of vacuum fluctuations, and it is these zero-point oscillations themselves that become modified and hence alter the emission rate. If one reasons along these lines, then it is easy to become convinced that cavity-induced changes in atomic radiation rates are of a purely quantum electrodynamical origin—a direct consequence of the existence of vacuum field fluctuations.(2)

However, using A. O. Barut's self-field approach to QED,(3) it is possible to show that spontaneous emission can be thought of as being triggered by Fourier components of the atom's own self-field, rather than by external vacuum fields. In this way the effect of a cavity is to alter the mode density of the atom's radiation reaction field and therefore change the emission rate.(4) In fact, one can use a classical model of an atom consisting of a harmonically oscillating charge pair—and upon including the effects of radiation reaction, one obtains the correct results, at least in the case of a dipole near a single-plane mirror.(5) It is easy to see from Fermi's golden rule, Eq. (1), that cavity modifications of dipole radiation rates should be essentially a classical phenomenon, since the effect arises from the changes in the mode density ρ, and this is the same classically or quantum electrodynamically. With this in mind, we shall now give an explicitly classical method for computing the cavity effect on atomic emission rates. We shall apply the result to a single dipole between parallel, perfectly-reflecting mirrors and show that we obtain the usual factor obtained in cavity QED. Then we shall discuss the results of a manifestly classical experiment in the radio-frequency regime that explicitly confirm these predictions.

2. CLASSICAL THEORY OF DIPOLE RADIATION IN A CAVITY

We shall confine our discussions here to a hollow cavity whose walls are composed of perfectly conducting material. This restriction to an empty cavity is not necessary, and in fact it is possible to treat a cavity filled with a possibly inhomogeneous dielectric medium in order to discuss the effect of a constant dielectric[6] or a periodically varying dielectric[7] on atomic emission rates.

We assume a normal mode decomposition of the vector potential $\mathbf{A}$ in the cavity as

$$\mathbf{A}_{\mathbf{k}}(\mathbf{r}, t) = \mathbf{a}_{\mathbf{k}}(\mathbf{r})\, e^{-i\omega_{\mathbf{k}} t} \tag{2}$$

where the normal mode functions $\mathbf{a}_{\mathbf{k}}(\mathbf{r})$ obey the Helmholtz equation

$$\nabla \times \nabla \times \mathbf{a}_{\mathbf{k}}(\mathbf{r}) - \frac{\omega_{\mathbf{k}}^2}{c^2}\, \mathbf{a}_{\mathbf{k}}(\mathbf{r}) = 0 \tag{3}$$

for the hollow of the cavity. If we denote the conducting surface enclosing the cavity by S, then the boundary conditions $\mathbf{E}_{\|}|_S = \mathbf{H}_{\perp}|_S$ for the tangential electric and normal magnetic fields, respectively, imply for normal mode functions $\mathbf{a}_{\mathbf{k}}(\mathbf{r})$, the conditions

$$\mathbf{a}_{\|}|_S = 0 \tag{4a}$$

$$\left.\frac{\partial \mathbf{a}_{\perp}}{\partial n}\right|_S = 0 \tag{4b}$$

where $\mathbf{a}_{\|}$ and $\mathbf{a}_{\perp}$ are transverse and normal components of $\mathbf{a}$ with respect to the conducting surface S, and $\mathbf{n}$ is an inwardly directed unit normal to that surface. For a reasonably behaved, simply connected region R enclosed by S, the Helmholtz equation (3) together with the boundary conditions (4) are sufficient to determine the modal functions $\mathbf{a}_{\mathbf{k}}(\mathbf{r})$. It is this modal decomposition that fixes the mode density ρ in Fermi's golden rule, Eq. (1), and so all the information needed to compute the transition rate must be contained in the solution to the boundary value problem, Eqs. (3) and (4). To show this explicitly in a classical context, we shall work in the Coulomb gauge which, in the absence of free charges, implies

$$\varphi = 0 \tag{5a}$$

$$\nabla \cdot \mathbf{A} = 0 \tag{5b}$$

for the scalar and vector potentials, respectively. In this gauge, then, $\mathbf{A}$ is completely transverse. The wave equation for the transverse electromagnetic dyadic Green's function $\overleftrightarrow{\mathbf{G}}$ can be written[7]

$$\nabla_{\mathbf{r}} \times \nabla_{\mathbf{r}} \times \overleftrightarrow{\mathbf{G}}(\mathbf{r}, t; \mathbf{r}', t') + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\overleftrightarrow{\mathbf{G}}(\mathbf{r}, t; \mathbf{r}', t') = \frac{4\pi}{c}\,\delta(t - t')\,\delta_{\perp}(\mathbf{r} - \mathbf{r}') \tag{6}$$

where $\delta_{\perp}$ is the usual transverse δ function. The Green's function tells us how the transverse vector electromagnetic field propagates in the cavity.

Since the normal mode functions $\mathbf{a}_{\mathbf{k}}(\mathbf{r})$ form a complete set, we may expand the Green's function in terms of them to get[7]

$$\overleftrightarrow{\mathbf{G}}(\mathbf{r}, t; \mathbf{r}', t') = c^2 \theta(t - t') \int d^3k\, \mathbf{a}_{\mathbf{k}}^*(\mathbf{r}')\, \mathbf{a}_{\mathbf{k}}(\mathbf{r})\, \frac{\sin[\omega_{\mathbf{k}}(t - t')]}{\omega_{\mathbf{k}}} \tag{7}$$

where c is the speed of light in a vacuum and $\theta(t)$ is a unit step function that explicitly enforces the causality of a retarded radiation field. It is clear, then, from Eq. (7) for the Green's function, that $\overleftrightarrow{\mathbf{G}}$ contains all the information on the cavity modal structure through its dependence on the $\mathbf{a}_{\mathbf{k}}$'s.

Let us now consider a localized radiating probe current source $\mathbf{J}(\mathbf{r}, t)$ that is introduced into the cavity at a point $\mathbf{r}$. (We assume that $\mathbf{J}$ is localized enough so that it adds only a negligible perturbation to the mode functions $\mathbf{a}_{\mathbf{k}}$.) The radiated power output $P(t)$ of this current is the rate at which it works against any surrounding electric field $\mathbf{E}$; hence,

$$P(t) = -\int_V dV\, \mathbf{J}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, t) \tag{8}$$

where the volume of integration needs to be large enough to include all of nonzero $\mathbf{J}$. The idea is that $\mathbf{E}$ is the field produced by $\mathbf{J}$ itself—a self-field, if you will—at earlier times that has consequently undergone multiple reflections off the conductor walls. Since $\varphi = 0$, we have $\mathbf{E} = -(1/c)\, \partial\mathbf{A}/\partial t$, and if we solve the wave equation for $\mathbf{A}$ in terms of $\mathbf{J}$,

$$\mathbf{A}(\mathbf{r}, t) = \frac{4\pi}{c} \int_{-\infty}^{\infty} dt' \int_V dV'\, \overleftrightarrow{\mathbf{G}}(\mathbf{r}, t; \mathbf{r}', t') \cdot \mathbf{J}(\mathbf{r}', t') \tag{9}$$

then we may write the power $P(t)$ in terms of $\mathbf{J}$ and $\mathbf{a}_{\mathbf{k}}$ using the expansion, Eq. (7). Taking the total energy U radiated up until time t as $U(t) \equiv \int dt'\, P(t')$ then, after some manipulation,[7] we have

$$U(t) = \pi \int d^3k \left| \int_{-\infty}^{t} dt' \int_{V'} dV'\, \mathbf{J}(\mathbf{r}', t') \cdot \mathbf{a}_{\mathbf{k}}(\mathbf{r}')\, e^{-i\omega_{\mathbf{k}} t'} \right|^2 \tag{10}$$

This formula shows precisely how energy accumulates in the field as a function of the source current $\mathbf{J}$ and the normal modes $\mathbf{a}_{\mathbf{k}}(\mathbf{r})$.

We choose now $\mathbf{J}$ for a point dipole located at $\mathbf{r}$ and oscillating at ω_0 as

$$\mathbf{J}(\mathbf{r}', t') = \sqrt{2}\, \omega_0 \boldsymbol{\mu} \cos(\omega_0 t')\, \theta(t')\, \delta(\mathbf{r}' - \mathbf{r}) \tag{11}$$

which is normalized so that the time average

$$\left\langle \left| \int dV'\, \mathbf{J}(\mathbf{r}', t') \right|^2 \right\rangle_{t'} = \omega_0^2 \mu^2 \tag{12}$$

where $\boldsymbol{\mu}$ is the dipole moment. With this current, then, the integrations in Eq. (10) for the energy $U(t)$ may be performed, and using $P(t) \equiv U'(t)$ we can show that the steady-state power is

$$P_\infty \equiv \lim_{t \to \infty} P(t)$$
$$= \pi^2 \omega_0^2 \mu^2 \int d^3k \, |\mathbf{a}_\mathbf{k}(\mathbf{r}) \cdot \hat{\boldsymbol{\mu}}|^2 \, \delta(\omega_0 - \omega_\mathbf{k}) \tag{13}$$

for a point dipole at $\mathbf{r}$. This result is the classical version of an analogous formula for spontaneous emission rates in terms of normal mode functions derived quantum electrodynamically.[6] To isolate the effect of the cavity alone, let us first compute the free-space power output as a normalization factor. In empty space, the plane wave modes are

$$\mathbf{a}_\mathbf{k}(\mathbf{r}) = (2\pi)^{-3/2} \, e^{-i\mathbf{k} \cdot \mathbf{r}} \hat{\varepsilon}_\mathbf{k} \tag{14}$$

where $\hat{\varepsilon}_\mathbf{k}$ is a transverse polarization vector so that $\hat{\varepsilon}_\mathbf{k} \cdot \mathbf{k} = 0$. Inserting these modes into Eq. (13), we arrive at the usual result for a point dipole, namely,

$$P_\infty^{\text{free}} = \frac{c}{3} k_0^4 \mu^2 \tag{15}$$

upon summing our polarizations. Here, $k_0 \equiv \omega_0/c$. We shall use this factor, Eq. (15), to normalize Eq. (13) to obtain the cavity-correction factor to the radiation rate, that is,

$$p(\mathbf{r}) \equiv P_\infty^{\text{cavity}}(\mathbf{r})/P_\infty^{\text{free}}$$
$$= 3\pi^2 \frac{c}{k_0^2} \int d^3k \, |\mathbf{a}_\mathbf{k}(\mathbf{r}) \cdot \hat{\boldsymbol{\mu}}|^2 \, \delta(\omega_0 - \omega_\mathbf{k}) \tag{16}$$

From this equation, we can see that the cavity factor essentially is proportional to the quantity $|\mathbf{a}_{\mathbf{k}_0}(\mathbf{r}) \cdot \hat{\boldsymbol{\mu}}|^2$ and hence the minima and maxima of the atomic emission rate simply follow the nodes and antinodes, respectively, of the mode functions. We emphasize that the $\mathbf{a}_\mathbf{k}(\mathbf{r})$ are not quantized and they control the emission rate in a purely classical fashion. In the next section we shall apply Eq. (10) for the cavity-induced changes in the emission rate to the famous example of a dipole located between two mirrors.

3. DIPOLE RADIATION BETWEEN MIRRORS

We now present a manifestly classical derivation for the effect of parallel mirrors on an enclosed point dipole oriented parallel to the mirrors.

In several famous experiments, it has been shown that when the "dipole" is an atom, the emission process can be completely halted if the atomic transition frequency ω_0 is less than the cavity cutoff $\omega_c = \pi c/d$.[8] Often, this result is explained in terms of the lack of vacuum fluctuations of frequency ω_0 in the cavity to initiate the decay. Our result here indicates that the inhibition of radiation below the cutoff arises from a destructive effect between the dipole and its own field. To clinch the argument, we will present data from a radio-frequency experiment in which the total quenching of dipole emission is seen at a macroscopic scale where there can be no possibility of invoking vacuum fluctuations in the description of the phenomenon.

We position a point dipole $\boldsymbol{\mu}$ at a location z_0 between two infinite, perfectly conducting mirrors separated by a distance d. We consider only the case where $\boldsymbol{\mu}$ is parallel to the mirrors, for only in this orientation is complete quenching of the emission possible below cutoff. (See Fig. 1.) The boundary conditions, Eq. (4), on the Cartesian components of the mode functions $\mathbf{a}(\mathbf{r})$ become

$$a_x(0) = a_x(d) = 0 \tag{17a}$$

$$a_y(0) = a_y(d) = 0 \tag{17b}$$

$$\frac{\partial a_z(0)}{\partial z} = \frac{\partial a_z(d)}{\partial z} = 0 \tag{17c}$$

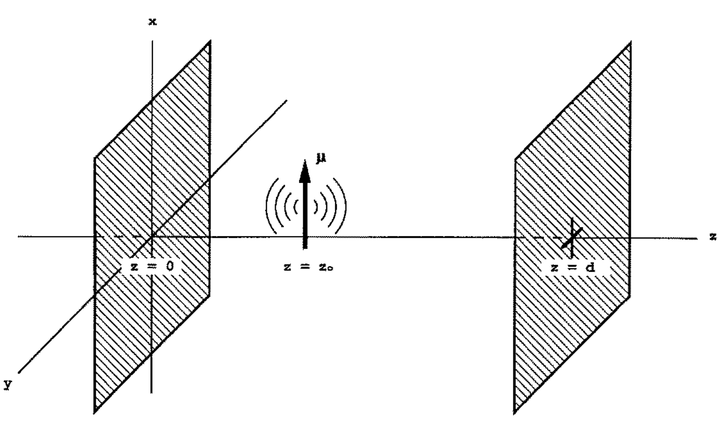

Fig. 1. We indicate here a radiating point dipole located between two parallel mirrors a distance z_0 from the left mirror located at $z = 0$. The right mirror is placed at $z = d$, the separation distance.

If we add to these the Coulomb gauge condition $\nabla \cdot \mathbf{a} = 0$ and the normalization condition

$$\int d^3k\, a^*_{\mathbf{k},i}(\mathbf{r})\, a_{\mathbf{k}',j}(\mathbf{r}) = \delta_{ij}\delta(\mathbf{k}-\mathbf{k}') \tag{18}$$

where i and j are polarization indices, then $\mathbf{a}_\mathbf{k}(\mathbf{r})$ for the cavity can be fixed to be

$$\mathbf{a}_\mathbf{k}(\mathbf{r}) = \frac{1}{2\pi}\sqrt{\frac{2}{d}}\left[\left(\hat{\mathbf{k}}_{\|} \times \hat{\mathbf{z}} - i\frac{k_n}{k}\hat{\mathbf{k}}_{\|}\right)\sin k_n z_0 - \frac{k_{\|}}{k}\hat{\mathbf{z}}\cos k_n z_0\right] e^{-i\mathbf{k}_{\|}\cdot\boldsymbol{\rho}} \tag{19}$$

where $\mathbf{k}_{\|}$ and $\mathbf{k}_n$ represent components of $\mathbf{k}$ parallel and normal to the mirrors, respectively, and $\boldsymbol{\rho}$ is the radial component of $\mathbf{r}$, transverse to the mirrors. The boundary conditions require discrete values for $k_n = n\pi/d$, for $n = 0, 1, 2, \ldots$, and we have the relation

$$k^2 = k_n^2 + k_{\|}^2 \tag{20}$$

If we now insert these parallel mirror modes, Eq. (19), into the cavity-correction formula for the power radiated by the dipole, Eq. (10), we obtain

$$p = \frac{3\pi}{2k_0 d} \sum_{n=1}^{[[k_0 d/\pi]]} [1 + (k_n/k_0)^2] \sin^2 k_n z_0 \tag{21}$$

where $[[x]]$ is "the greatest integer less than x." This is precisely the correction factor to the spontaneous emission rate obtained by Barton and Agarwal in a quantized mode expansion calculation,(2) by Milonni and Knight in a second quantized approach invoking the method of images,(9) or by Barut and Dowling using self-fields.(4) I re-emphasize here that this calculation leading to Eq. (21) has been totally classical. We now plot, in Fig. 2, $p = p(d, z_0)$ in terms of a fixed dipole emission wave number k_0 and a variable mirror separation d, and position coordinate between the mirrors, z_0. We use unitless mirror separation and dipole position parameters, $\delta \equiv d/(\lambda/2)$ and $\xi = z_0/d$ in the graph. In these units, the resonant modes of the cavity are given by $\delta = n$, and are seen in Fig. 2 as sharp "cliffs" in the power output. For separations below cutoff, $\delta < 1$, we see a flat valley of height zero that indicates the complete quenching of the emission regardless of the dipole position ξ. At $\delta = 1$, we see a position-dependent power output that rises and falls with antinode and nodes of the fundamental resonant mode. For $\xi = 1/2$, midway between the mirrors, p has a value of three that indicates a power output three times that of the free space value, $p = 1$. As we increase the mirror spacing $\delta = d/(\lambda/2)$, we

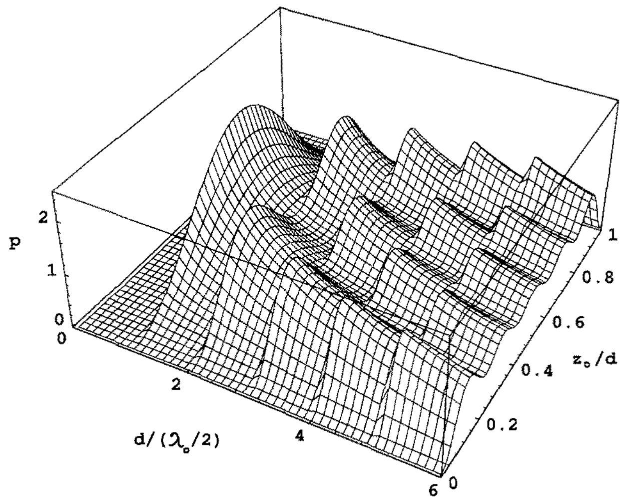

Fig. 2. We plot the theoretical, free-space-normalized, power output $p(d, z_0)$, Eq. (21), of a point dipole between mirrors as a function of mirror separation d and dipole location between the mirrors, z_0. We have fixed the dipole frequency at ω_0 and are using the unitless separation and position parameters, $\delta \equiv d/(\lambda_0/2)$ and $\xi \equiv z_0/d$, respectively, where λ_0 is the dipole radiation wavelength, $\lambda_0 = 2\pi c/\omega_0$. The quenching of the emission for $d < \lambda_0/2$ is clearly visible as the flat valley of height zero. (The plane $p = 1$ corresponds to the free space radiation rate.) We see sharp emission enhancements and suppressions at the resonance separations $d = n\lambda/2$, where $n = 1, 2, 3,...$, such that an integer number of half-wavelengths fit across the cavity. The oscillations of the power in the $\xi = z_0/d$ directions are due to the oscillations in the modal wave functions $|\mathbf{a}_{\mathbf{k}_n}(\mathbf{r})|^2$ that modulate the power output, as per Eq. (16).

see the resonant mode at $d = n\lambda/2$ appearing for $n = 1, 2, 3,...$. As before, the $\xi = z_0/d$ dependence tracks the higher harmonic resonant modal functions $|\mathbf{a}_{\mathbf{k}_n}(z_0)|^2$. In the next section we compare this result to a radio-frequency experiment.

4. A RADIO-FREQUENCY EXPERIMENT

Once it is acknowledged that the effect of parallel mirrors on point dipole radiation rates is manifestly classical in origin, there is no need to perform the experiment in microcavities[8] in order to see the change in the emission rates. So saying, a macroscopic radio-frequency version of the experiments in Ref. 8 has been carried out at the University of Alabama in

Huntsville by undergraduate students working with two 64 ft^2 sheets of aluminized building insulation material for the mirrors, and a half-wave dipole antenna radiating at a wavelength of $\lambda \cong 30$ cm for the "atom."[10] This experiment, as a mock-up of the famous cavity QED apparatuses, has the pedagogical merit of inculcating an understanding of the principles involved in the cavity-induced modification of emission rates. Our experiment also drives home the fact—in a very "hands-on" fashion—that the phenomenon is truly classical in nature. Vacuum fluctuations just are not viable as an explanation of the results when the mirror separation is on the order of tens of centimeters.

The basic idea of one portion of the experiment is to support the mirrors vertically and to suspend the dipole from the ceiling with string—midway between, and parallel to, the mirrors. Hence, we take $z_0 = d/2$ in Figs. 1 and 2 and also in the power output formula, Eq. (21). A signal

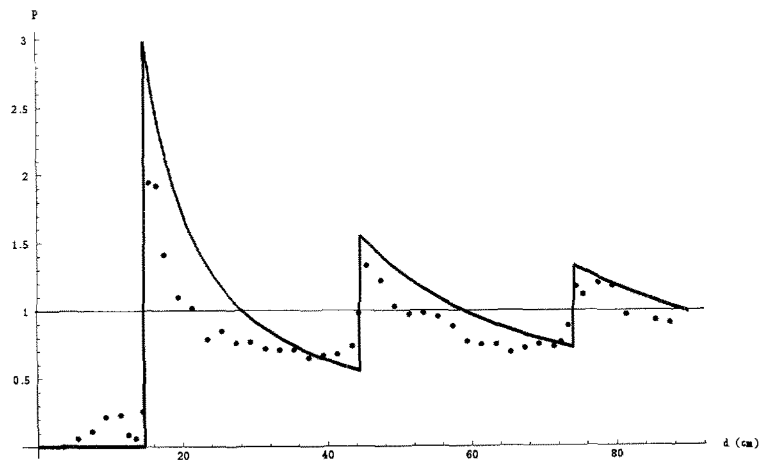

Fig. 3. Here we plot, as a thick solid line, the theoretical curve for the free space normalized power output $p(d, z_0)$, Eq. (21), for a dipole centered between mirrors at $\xi = z_0/d = 1/2$. The mirror separation d is given here in centimeters. This plot then corresponds to a vertical slice through the surface in Fig. 2 on the plane $z_0 = d/2$. The black dots here are data points taken from a radio-frequency experiment using a half-wave dipole oriented parallel to the mirrors and radiating at a wavelength of $\lambda_0 \cong 30$ cm. The mirrors are composed of vertically mounted, 64 ft^2 sheets of aluminized insulation material whose separation was adjusted by hand. We see here that there is excellent agreement—in particular we see the strong suppression of emission for $d < \lambda/2$ and the enhancements and suppressions at $d = n\lambda/2$, with $n = 1, 2, 3,...$. The thin horizontal line at $p = 1$ corresponds to the free space power output. The experiment is a macroscopic mock-up of several cavity QED experiments and illustrates the essentially classical nature of the effect.

generator sends power down two coaxial cables to the antenna, and then a directional coupler is used to determine the actual amount of power radiated. To normalize the data to free space, we measured the power radiated when the mirrors were removed. Then, to compare Eq. (21) to the experiment, we take $p_{\text{exp}} = $ (*power radiated between mirrors*)/(*power radiated in free space*). In Fig. 2, the plane $z_0 = d/2$, or equivalently, $\xi = 1/2$, slices vertically through the middle of the $p(d, z_0)$ surface. The resulting theoretical curve, $p(d, d/2)$, is plotted in Fig. 3, along with data points taken at a dipole radiation wavelength of $\lambda \cong 30$ cm. The students would move the mirrors back and forth, varying d, allowing p_{exp} to be determined. These data points for one run are superposed on the theoretical curve in Fig. 3, and we see excellent agreement.[10] In particular, the emission quenching for $d < \lambda/2$ is clearly evident, and the antinodal enhancements at $d \cong n\lambda/2$, for odd n, are also prominent. Resonances for even n correspond to the dipole at $z_0 = d/2$ being at a modal node, and we see emission suppression below the free space value of $p = 1$ that is indicated by the thin horizontal line.

5. SUMMARY AND CONCLUSIONS

We have given a general classical method of computing the effect of a conducting cavity on the radiation rate of a localized current source, Eq. (10). Then we applied this formalism to the question of point dipole emission between infinite parallel mirrors, and we obtain the same correction factor used in cavity QED experiments and usually derived from quantum electrodynamical considerations. This calculation is a quantitative complement to a more qualitative treatment of the result as a problem in classical antenna theory, as developed in Ref. 11. This present work proves, then, that the effect of spontaneous emission changes in cavities is about as classical as one can possibly get. To illustrate this in a striking fashion, we compare the theory to a radio-frequency mock-up of the famous cavity QED experiments of Ref. 8 and we reproduce the microcavity results nicely—refuting once and for all the notion that one must somehow invoke electromagnetic zero point fluctuations to explain this phenomenon.

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