QED BASED ON SELF-FIELDS: CAVITY EFFECTS

Jonathan P. Dowling

Max-Planck-Institute für Quantenoptik
D-8046 Garching
Federal Republic of Germany

I. INTRODUCTION

The second quantization of the electromagnetic field was performed for one of the first times in a seminal paper by Dirac in 1927;[1] this step is usually considered to have been the dawn of QED. Although the second quantization hypothesis allowed Dirac to derive the Einstein A coefficient of spontaneous emission—further results were not forthcoming until approximately 20 years later when regularization and renormalization schemes were invented to treat the various singular expressions which arose in the theory.[2] One particular singularity, however, seems to resist being swept under the rug and continually keeps crawling back out again. It is well known that the second quantization procedure predicts that the electromagnetic vacuum contains a zeropoint energy corresponding to $\frac{1}{2}\hbar\omega_k$ per normal mode k of the field. This gives rise to a divergent vacuum energy density,[3] which one may theoretically renormalize away by demanding that the photon destruction and creation operators a and $a^\dagger$ be normally ordered, i.e. with the destruction operators all to the right. From the standpoint of general relativity this seems unsatisfactory, however. The stress energy tensor $T^{\mu\nu}$ has a physical connection to the spacetime metric $g^{\mu\nu}$ via the Einstein field equations. A divergent vacuum $T^{\mu\nu}$ would imply an infinite curvature for the universe, and such a curvature can not be removed simply by performing some sort of transfinite shift of the energy scale. An infinite vacuum energy corresponds to an infinite cosmological constant $\Lambda_{\text{thy}} = \infty$. Yet observations of the motion of distant galaxies puts an upper limit on the cosmological constant of $\Lambda_{\text{exp}} < 10^{-56}\ \text{cm}^{-1}$. This, the famous *Cosmological Constant Problem*,[4] casts doubt on the physical reality of vacuum field fluctuations. Nevertheless, many of us apparently would like to have our vacuum and eat it too. There is a longstanding tradition in QED to take the existence of the zeropoint fluctuations as real things, and to use them to carry out calculations of radiative effects. By coupling an electron to the vacuum fluctuations one may obtain a satisfactory account of the Lamb shift and spontaneous emission—although one can not get easily a sensible value for $g-2$ using this method.[5—8] This process of coupling the electron to the zeropoint field as a method of calculation we shall call nonrelativistic QED. Casimir forces and apparatus contributions to such things as spontaneous emission, the Lamb shift, and $g-2$ can also be calculated in this fashion.[9—11] Can nonrelativistic QED be trusted? If taken seriously it gives good results for for spontaneous emission and the nonrelativistic Lamb shift, cryptic or ambiguous results for $g-2$, and complete nonsense for the cosmological constant. How can we be comfortable with any prediction of this theory in the absence of a fully relativistic calculation to back it up? In standard QED one second quantizes the free electromagnetic field separately from the electronic field, and only then does one couple the two entities. Is this procedure valid? As Einstein has warned us:

> I feel that it is a delusion to think of the electrons and the fields as two physically different, independent entities. Since neither can exist without the other, there is only *one* reality to be described, which happens to have two different aspects; and the theory ought to recognize this from the start instead of doing things twice.[12]

Perhaps the vacuum field is not physically real after all—in fact some argue that it can not be—perhaps it is only a mathematical artifact of the second quantization procedure.[12,13] General relativity also presents us with a second troubling problem inherit in the quantum field theoretic notion of the vacuum. The normal mode decomposition of the electromagnetic field is unique only in Minkowski space. In curved spacetime this is not so, and hence different observers will see different vacua. This conclusion has as its consequence such phenomena as Hawking radiation from a black hole, and the Unruh effect in which an accelerating detector registers a thermal bath of photons. This is quite distressing—if an inertially moving detector and a uniformly accelerating detector are near each other in spacetime, the inertial one sees nothing, while the accelerating one sees a Planck distribution of photons. If these photons are real, why doesn't the inertial detector see them too? Such paradoxes have led P. C. W. Davies to conclude that the concept of 'particle' (in this case 'photon') breaks down in curved spacetime.[14] This is pretty strong stuff! The acceptance of standard quantum field theory implies that the particle notion is nonsense in curved space. Can such a conclusion be avoided? Yes, it is possible to rescue the notion of 'the photon' if we abandon the quantum field notion of 'the vacuum.'

It is standard folklore to believe that radiative effects such as spontaneous emission and the Lamb shift are caused by the interaction of the electron with zeropoint fluctuations. If we dispose of the vacuum fluctuations, what then is the causative agent behind these radiative corrections? There exist perfectly respectable classical analogs of spontaneous emission and the Lamb shift. A harmonically bound charge will exhibit a line broadening and a level shift if the equation of motion includes radiation reaction—and no field fluctuations are needed to explain this result. Is it somehow possible to take the classical theory of radiation reaction and generalize it to a quantum mechanical setting? Schrödinger was one of the first to point out that the back reaction of the electron's own field on itself must be added to the Schrödinger equation in order to have an equation of motion which could be considered complete.[15] Fermi also tried something along this line.[16] By inserting a classical-like radiation reaction term into the Schrödinger equation, he arrived at what essentially was the neoclassical theory of Crisp and Jaynes.[17] This approach yields the correct Einstein A coefficient—but a nonexponential "chirped" decay profile. (Recent work seems to indicate that the nonphysical decay law of the neoclassical theory is a mathematical error arising from an invalid application of the superposition principle in a nonlinear theory.[18]) The neoclassical approach seems a bit *ad hoc*, and it turns out that there is a more natural and complete way to include radiation reaction in quantum mechanics.

In 1938 Dirac was able to derive the classical, covariant Abraham-Lorentz equation of motion for a charge which includes radiation reaction.[19] In particular, he had to assume that the electromagnetic potential A_μ surrounding a charge is symmetric in the retarded and advanced fields to arrive at his result. In 1945, Wheeler and Feynman elaborated on the work of Fokker, Tetrode, and Schwarzschild to produce the *Absorber Theory* or *Action at a Distance Electrodynamics.*[20—22] The idea here is that one can produced all of electrodynamics—Maxwell's equations and the Abraham-Lorentz-Dirac (ALD) equation of motion—from a single action principle *if* one assumes that the action density is symmetric with respect to future and past, or, equivalently, in the retarded and advanced fields. It is well known that Wheeler and Feynman never produce a quantum version of this theory. Süssman has produced a fully second quantized version of action at a distance electrodynamics, from which he was able to arrive at the A coefficient.[21] Barut and his coworkers, however, have gotten this coefficient—and more—with intermediate versions of the theory which are not second quantized, but rather which extend the action principle of Wheeler and Feynman to include Schrödinger, Pauli, and Dirac action principles, rather than just the classical.[18,23—25] The contention is that the covariant inclusion of radiation reaction is to be done instead of—not in addition to—second quantization. This approach to QED has led to correct results for relativistic accounts of spontaneous emission,[26] the Lamb shift,[27] the $g-2$ anomaly,[28] and vacuum polarization.[29] In the nonrelativistic approximation, spontaneous emission and the Lamb shift,[25] $g-2$ of the electron,[8] cavity QED effects,[30—32] and the Unruh effect[33] have been calculated. In this paper we shall summarize some of the cavity results as well as the Unruh effect calculation. It should be mentioned that Casimir forces can be equally well derived in the self-field approach, even though there are no vacuum fluctuations.[31,34] All phenomena which hitherto were thought to be caused by zeropoint energy can apparently be explained in terms of self-fields. In fact Jaynes has shown[12] that the radiation reaction spectrum over the linewidth of an atom is equal to the vacuum fluctuation spectrum. In the self-field approach the vacuum field is assumed to be zero for all moments of the correlation functions. For example, to trigger spontaneous emission, an atom produces a radiation reaction field on itself in just the right amount to cause a decay. Compare this to the quantum field philosophy in which one must fill the entire universe with an infinite density zeropoint energy in order to get spontaneous emission for a single atom. Since in self-field QED the vacuum field is identically zero, there is no longer a cosmological constant problem. Self-field theory *predicts* a cosmological constant of zero—in excellent agreement with experiment. The self-field approach also solves the paradox that usual QED leads to in curved space. It has been shown that the Unruh effect can be calculated in self-field theory, and the result is precisely the same as in standard QED.[33] An accelerating detector responds *as if* bathed with thermal photons, whereas the inertially moving detector sees nothing. But now the conclusion is different: The thermal photons are not real, but rather the accelerating agent directly stimulates the self-field of the detector—the causative agent of spontaneous emission—and forces the atom into a superposition of states which corresponds to a thermal distribution. This neatly accounts for the fact that a nearby inertial detector sees no photons, and hence rescues the concept of photon as a particle. The cost of saving the photon is the loss of a dynamic, interactive vacuum. Zeropoint fluctuations in empty space are perhaps only a useful fiction, from the self-field frame of mind. Even the nonrelativistic calculation of $g-2$ in the self-field calculation unambiguously gives the correct sign, in contradistinction to a standard QED vacuum field calculation.[5,7,8]

It should be mentioned that in the context of standard QED there seems to be a dual relation between the vacuum fluctuation and the radiation reaction interpretations.[6,12,35—39] The consensus here appears to be that, *within the framework of standard QED*, both interpretations are required for a cogent theory of spontaneous emission. There remains the possiblity that a modified version of QED might not contain zeropoint fluctuations at all—that they might only be a mathematical subterfuge introduced by the second quantization prescription. It is proposed that self-field QED, so far at least to order α, is such a theory.

II. CAVITY EFFECTS IN QED

In fully relativistic QED, the freespace Feynman propagator is a globally defined Green's function for the electromagnetic field. As such, its structure will depend on the environment and the external boundary conditions imposed on the field. Thus the presence of conducting surfaces, for instance, in the neighborhood of an atom will alter such radiative effects as spontaneous emission, the Lamb shift, $g-2$, etc. whose calculations depend explicitly on the form of the propagator. In nonrelativistic QED one demands that the vacuum field obeys some appropriate boundary conditions, and then one couples the atom to the modified vacuum in order to calculate for apparatus induced changes to the usual freespace radiative corrections. In self-field QED the self-field A_μ^{self} of the electron is eliminated from the total action through use of the *same* Feynman's Green function used in standard QED. Hence, one expects similar boundary corrections as in standard QED, but now the understanding is that it is the radiation reaction field of the atom—and not the zeropoint field—which is adapting itself to a new environment.

For example, it is experimentally well verified that the spontaneous emission rate of an atom between parallel conducting plates can be suppressed nearly completely.[40,41] In standard QED the interpretation goes something as follows. Consider a two level atom of frequency ω_o. In freespace the atom finds all modes of the vacuum available to it, including that which also has frequency ω_o which is capable of stimulating spontaneous emission. Suppose now the atom is placed between parallel plates whose spacing L is too small to support the vacuum mode corresponding to ω_o. This occurs when $L < \lambda_\text{o}/2 := \pi/\omega_\text{o}$. In this case, even the zeroth harmonic corresponding to one half of a wavelength of a standing wave can not fit between the plates, and so the ω_o mode of the vacuum vanishes and spontaneous emission turns off. Those accustomed to this account of the phenomenon may find it difficult to believe that a theory without zeropoint fluctuations can produce the same result. Let us see how. Reconsider the two level atom in freespace. The atom is exposed to its own radiation reaction field, which when Fourier analyzed, contains all the same frequency components found in the vacuum field before. Hence there is a Fourier component of the self-field of frequency ω_o at hand to trigger spontaneous emission of our atom. Now between parallel mirrors, each Fourier component of the reaction field must separately obey the new boundary conditions. The condition $L < \lambda_\text{o}/2$ will completely wipe out all Fourier components of the self-field with frequency of ω_o or lower—and spontaneous emission will cease.

III. THE SELF-FIELD ACTION FORMALISM

In analogy to classical, action at a distance electrodynamics we wish to specify an action $W := \int dx\, w(x)$ which has as its Euler-Lagrange equations of motion Maxwell's equations for the electromagnetic (EM) field, and for the particle, an equation which includes radiative effects. We assume that the action density $w(x)$ consists of a free particle term w_o, a free field term w_f, and an interaction term w_i. Hence, the general form of w is, with the convention $\hbar = c = 1$,

$$\begin{aligned} w &= w_\text{o} + eA_\mu j^\mu + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &=: w_\text{o} + w_\text{i} + w_\text{f} \end{aligned} \tag{1}$$

This expression at first appears to be the usual semiclassical action density. In standard semiclassical quantum theory, however, it is assumed that the A_μ which appears here is an external field from sources assumed to be at infinity. In particular A_μ would not include the self-field of the charge. But, according to Schrödinger,[15] this self-field *must* be included if one hopes to have a complete description of radiation reaction. To account for the self-field in a covariant fashion we make the *ansatz* that the EM potential surrounding the charge can be split into an external and self-field contribution as

$$A_\mu = A_\mu^{\text{ext}} + A_\mu^{\text{self}} \tag{2}$$

where A_μ^{ext} and A_μ^{self} obey the homogeneous and inhomogeneous Maxwell's equations, respectively, in a localized interaction region surrounding the charge. These equations are

$$\partial_\nu F_{\text{ext}}^{\mu\nu} = 0 \tag{3a}$$

$$\partial_\nu F_{\text{self}}^{\mu\nu} = e\, j^\mu \tag{3b}$$

The general solution to the nonhomogeneous equation (3*b*) can be written with a Green's function $D_{\mu\nu}(x-y)$ as[20]

$$A_\mu^{\text{self}}(x) = e \int dy\; D_{\mu\nu}(x-y)\, j^\nu(y) \tag{4}$$

where $x := x_\mu$, $y := y_\mu$, and $dy := d^4y$. Thus the integral is carried out over all Minkowski space. Equation (4) is the single most important feature of the self-field approach to QED, and hence bears a brief discussion. Expression (4) allows one to eliminate the self-field from the action, and hence from the equations of motion, in a covariant fashion. If we allowed our interaction region to include the entire universe, then $A_\mu = A_\mu^{\text{self}}$ alone and there would no longer be any A_μ^{ext}. Hence, as is apparent from the form of this equation, all electromagnetic potentials A_μ have their origin in some source current. As a consequence, electromagnetic fields do not exist independent of the sources that produce them. Considerations such as these have led to revival of the ***Schrödinger Interpretation of Quantum Mechanics*** in which the electron wave function is viewed as an actual distribution of electronic charge, as opposed to the usual probabilistic interpretation.[42—44] There is no such thing as a 'free' EM field, and consequently no such thing as a vacuum EM field. The self-field approach predicts that the vacuum is empty of electromagnetic energy, and that the EM vacuum contributes zero to the cosmological constant. A common criticism of self-field QED is that since the EM field is treated classically, one can not hope to obtain a complete theory of QED, since the field is not quantized. But from equation (4) it is clear that A_μ^{self} will be classical only if the source current j_μ is classical. On the other hand, if j_μ corresponds to a Schrödinger, Pauli, or Dirac current, then clearly A_μ^{self} will have quantum mechanical properties also, which it inherits from the quantum mechanical source. It is our position that the second quantization of the EM field is perhaps an unnecessary duplication of what is already contained in this expression (4). Maybe one does not have to second quantize A_μ if it is *already* quantized, inasmuch as it always exhibits quantum properties due to the quantized source which produced it. As a longstanding critic of the second quantization procedure, E. T. Jaynes tells us:

> One can hardly imagine a better way to generate infinites in physical predictions than by having a mathematical formalism with $(\infty)^2$ more degrees of freedom than are actually used by Nature.[12]

In the action density w of equation (1) we have not specified the form of $w_\circ$ or j_μ. Actually, after specifying $w_\circ$, the requirement that the variation of the total action

W with respect to A_μ yields an extremum give us Maxwell's equations and identifies the form of j_μ. The four cases of interest to us here are

1. *Classical Action Density and Current*

$$w_i = m\dot{z}^2 - eA_\mu \dot{z}^\mu \tag{5a}$$

$$j^\mu = \int d\tau \; e\dot{z}^\mu \; \delta(x - z(\tau)) \tag{5b}$$

2. *Schrödinger Action Density and Current*

$$w_i = \psi^* \Big[\frac{1}{2m}(\overleftarrow{\nabla} + ie\mathbf{A}) \cdot (\overrightarrow{\nabla} - ie\mathbf{A}) + eA_\circ - i\frac{\partial}{\partial t}\Big]\psi \tag{6a}$$

$$j^\mu = \psi^* \Big[1, \frac{1}{2mi}\overleftrightarrow{\nabla} - \frac{e}{m}\mathbf{A}\Big]\psi \tag{6b}$$

3. *Pauli Action Density and Current*

$$w_i = \phi^* \Big\{\frac{1}{2m}\big[(\overleftarrow{\nabla} + ie\mathbf{A}) \cdot \sigma\big]\big[\sigma \cdot (\overrightarrow{\nabla} - ie\mathbf{A})\big] + eA_\circ - i\frac{\partial}{\partial t}\Big\}\phi \tag{7a}$$

$$j^\mu = \phi^* \Big[1, \frac{1}{2mi}\overleftrightarrow{\nabla} + \frac{1}{2m}(\overleftarrow{\nabla} \times \sigma - \sigma \times \overrightarrow{\nabla}) - \frac{e}{m}\mathbf{A}\Big]\phi \tag{7b}$$

4. *Dirac Action Density and Current*

$$w_i = \overline{\Psi}\big[\gamma^\mu(i\partial_\mu - eA_\mu) - m\big]\Psi \tag{8a}$$

$$j^\mu = \overline{\Psi}\gamma^\mu\Psi \tag{8b}$$

Variation of W with respect to z_μ, ψ, ϕ, or Ψ yields, respectively, the ALD, Schrödinger, Pauli, or Dirac equations of motion. The classical version is essentially the absorber theory of Wheeler and Feynman. The Schrödinger and Pauli actions produce nonrelativistic versions of self-field QED—without and with spin respectively. Finally, in the Dirac case, we have a fully covariant theory of electronic motion which includes radiation reaction. One can then treat this final relativistic version as a possible candidate for a complete theory of QED—perhaps equivalent or dual to the usual second quantized version.

IV. SPONTANEOUS EMISSION BETWEEN MIRRORS

To illustrate the self-field methodology, we now sketch a calculation of how the Einstein A coefficient for spontaneous emission changes between parallel, perfectly conducting mirrors. For this problem it is sufficient to consider the Schrödinger action density and corresponding current found in equation (6). The total action density w of equation (1) then reduces to

$$\begin{aligned} w =& \psi^* \Big[\Big(\frac{1}{2m}\nabla^2 - i\frac{\partial}{\partial t} + eA_\circ^{\text{ext}}\Big) + \Big(\frac{ie}{2m}\mathbf{A}^{\text{self}} \cdot \nabla\Big) + \Big(\frac{e}{2}A_\circ^{\text{self}}\Big)\Big]\psi \\ =:& \psi^* \Big[(H_1) + (H_2) + (H_3)\Big]\psi \\ =:& w_1 + w_2 + w_3 \end{aligned} \tag{9}$$

To arrive at this relation we have used the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ and the weak field approximation $|\mathbf{A}^{\text{ext}}| \approx 0$. We have also neglected terms of order α^2 or higher, where the fine structure constant $\alpha := e^2/4\pi$ in our units. Finally, to reproduce a hydrogen-like atom we took $\mathbf{A}^{\text{ext}} = 0$ and $A_\circ^{\text{ext}} = -Ze/r$. If we were to set $A_\mu^{\text{self}} \equiv 0$

in equation (9) we would arrive at just the usual Schrödinger equation for hydrogen. However, since $A_\mu^{\rm self}$ is proportional to the electron current via equation (4), we can not consistently set it equal to zero unless $j_\mu \equiv 0$ over all spacetime—in which case we have no electron! Hence, by *reductio ad absurdum* it is clear that the inclusion of $A_\mu^{\rm self}$ in the Schrödinger equation is *required.* Consequently, the Schrödinger action contains nonlinear, nonlocal terms of the generic form

$$W^{\rm self} = \frac{e^2}{2} \int\!\!\int dx\; dy\; j^\mu(x)\; D_{\mu\nu}(x-y)\; j^\nu(y) \tag{10}$$

implying a nonlinear, nonlocal, integro-differential Schrödinger equation. In freespace, the Green's function $D_{\mu\nu}$ in the Coulomb gauge has the form.

$$D_{ij}(x-y) = \frac{1}{(2\pi)^4} \int dk\; \frac{e^{-ik\cdot(x-y)}}{k^2 + i\epsilon} \left(\delta_{ij} + \hat{k}_i \hat{k}_j\right) \tag{11a}$$

$$D_{00}(x-y) = \frac{1}{(2\pi)^4} \int dk\; \frac{e^{-ik\cdot(x-y)}}{\lambda^2 + i\epsilon} \tag{11b}$$

$$D_{i0}(x-y) = D_{0i}(x-y) = 0 \tag{11c}$$

where $\lambda := |\mathbf{k}|^2$, $k^2 := k^\mu k_\mu$, and the $+i\epsilon$ in the denominator insures that the correct symmetry between retarded and advanced solutions to Maxwell's equations are obtained—a choice which is required if the action is to have an extremum.[22,—20] With this choice of Green's function, equation (4) can be written as

$$\mathbf{A}^{\rm self}(x) = -\frac{e}{(2\pi)^4} \int\!\!\int dy\; dk\; \frac{e^{-ik\cdot(x-y)}}{k^2 + i\epsilon} \Big[\mathbf{j}(y) - \hat{\mathbf{k}}(\hat{\mathbf{k}}\cdot\mathbf{j}(y))\Big] \tag{12a}$$

$$A_{\rm o}^{\rm self}(x) = \frac{e}{(2\pi)^4} \int\!\!\int dy\; dk\; \frac{e^{-ik\cdot(x-y)}}{\lambda^2 + i\epsilon} \rho(y) \tag{12b}$$

where ρ and $\mathbf{j}$ are the time and space components of the current j_μ as given in equation (6*b*). In our notation above we use $dy := d^4y$, $dk := d^4k$, and $\hat{\mathbf{k}} := \mathbf{k}/|\mathbf{k}|$. If this expression for $\mathbf{A}^{\rm self}$ is inserted into the action density w as given in expression (9), one can extract from the H_1 term a complex energy shift to level n which is given by

$$\begin{aligned} \mathcal{E}_1^{(n)} &= \frac{W_1^{(n)}}{2\pi} \\ &= \frac{2\alpha}{3\pi} \sum_m \omega_{nm}^3 |\mathbf{r}_{nm}|^2 \int_0^\infty \frac{d\lambda}{\omega_{nm} - \lambda} - \frac{2\alpha i}{3} \sum_{\substack{m \\ m<n}} \omega_{nm}^3 |\mathbf{r}_{nm}|^2 \\ &:= \delta E_n - i A_n \end{aligned} \tag{13}$$

where $\omega_{nm} := E_n - E_m$ and the $\mathbf{r}_{nm}$ are the usual matrix elements of the atomic position operator $\mathbf{r}$. This result is just Bethe's nonrelativistic Lamb shift formula, and also the Einstein A coefficient, which appears here as a damping term. A complex energy shift is interpreted as a line broadening in the usual fashion, giving rise to spontaneous emission.[18,25]

The self-field $A_\mu^{\rm self}$ we see is a globally defined quantity whose form depends on the Green's function which in turn depends on the environment. Clearly, if the Green's function is changed, the Lamb shift and spontaneous emission rates—and in fact all radiative corrections—must change. The self-field must meet the newly imposed boundary conditions. It is easy to show that the Green's function between parallel mirrors can be constructed by the method of images. Suppose we place two perfectly

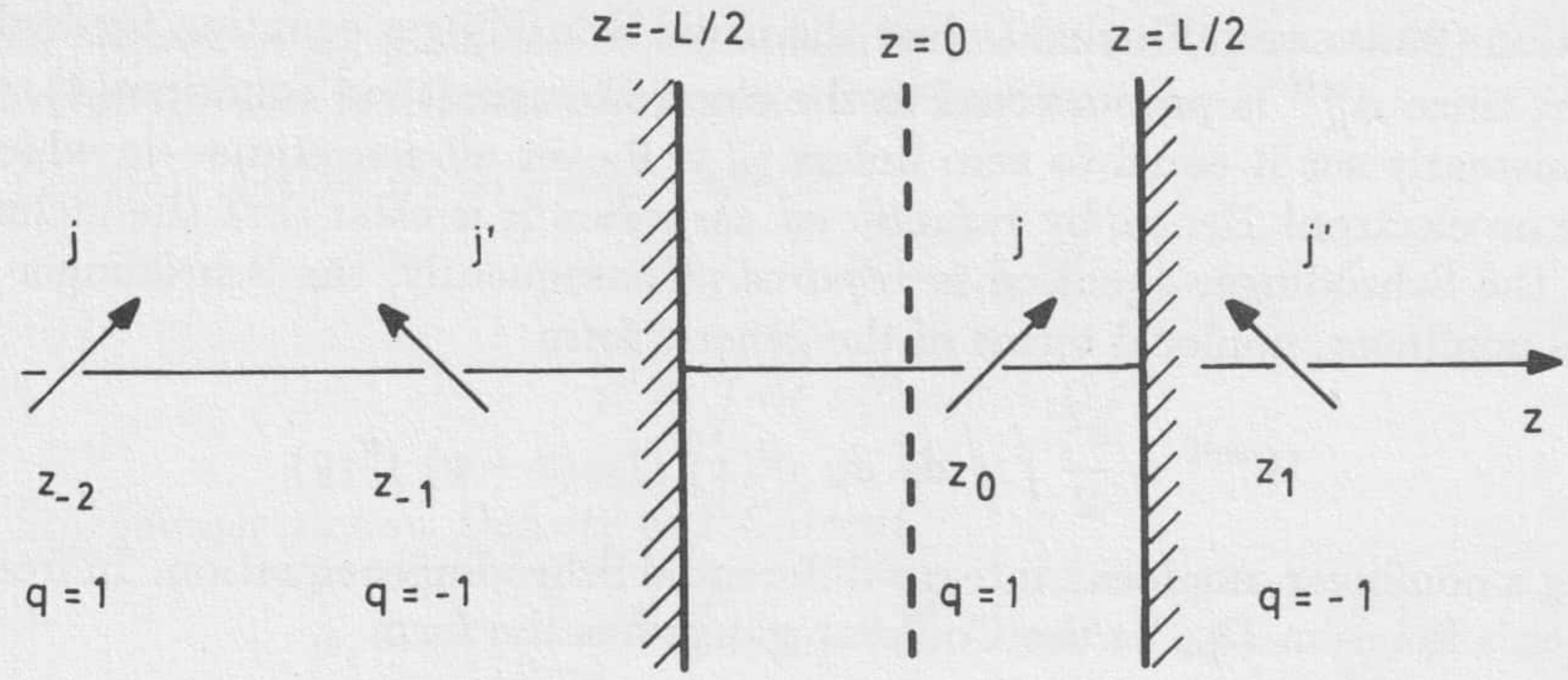

Fig. 1. A unit charge between parallel plates, located at $z_\circ$, and the resultant series of image charges of charge $(-1)^p$ located a $z_p := pL + (-1)^p z_\circ$, for $p = \pm 1, \pm 2, \pm 3 \ldots$

conducting, plane mirrors normal to the z axis at $z = \pm L/2$. A unit charge placed on the axis at $z = z_\circ$ will have image charges located at $z_p := pL + (-1)^p z_\circ$, where $p = \pm 1, \pm 2, \ldots$ (See Fig. 1) The total Green's function is then seen to be the infinite sum of translated freespace Green's functions, and it has the form

$$D_{\mu\nu}^{\text{mirrors}}(x - y) = \sum_{p=-\infty}^{\infty} D_{\mu\nu}^{\text{freespace}}(x - y_p' - z_p) \tag{14}$$

where

$$y_p' := \begin{cases} (y_\circ, y_1, y_2, y_3), & \text{if } p \text{ even} \\ (y_\circ, y_1, y_2, -y_3), & \text{if } p \text{ odd} \end{cases}$$

and $z_p := z_p^\mu := (z_p, 0, 0, z_p)$ is the image location in Minkowski space evaluated at the retarded time $t = z_p/c$.

After some work, we can show that the use of this Green's function gives rise to a complex energy shift similar to that seen in expression (13), but now in the limit that the plate spacing $L \ll c/A_\circ$,where $A_\circ$ is the freespace value the A coefficient becomes the following expression[30]

$$\begin{aligned} A_n =& \alpha \sum_{\substack{m \\ m<n}} \omega_{nm}^2 |\mathbf{r}_{nm}|^2 \sum_{p=1}^{[[\sigma_{nm}]]} \left\{ \left[(1 + \zeta_{nm}) + (1 - 3\zeta_{nm}) \left(\frac{p}{\sigma_{nm}} \right)^2 \right] \right. \\ & \left. - \left[(1 - 3\zeta_{nm}) + (1 + \zeta_{nm}) \left(\frac{p}{\sigma_{nm}} \right)^2 \right] \cos \left[\pi p \left(\frac{2z_\circ}{L} - 1 \right) \right] \right\} \end{aligned} \tag{15}$$

where $\sigma_{nm} := L\omega_{nm}/\pi$ and $\zeta_{nm} := |z_{nm}|^2/|\mathbf{r}_{nm}|^2$. The notation $[[x]]$ stands for the 'greatest integer less than x' function. This formula (15) agrees with the standard QED calculations of Barton,[45] Milonni and Knight,[46] Philpott,[47] and an experiment done by Hulet, Hilfer and Kleppner.[41] In particular—if we average formula (14) over the plate separation, assuming a uniform distribution of atoms between the plates, we obtain the A coefficient as a function of the plate spacing L as indicated in Fig. 2. As was mentioned earlier in this paper, for a plate spacing of $L < \lambda_\circ/2$, where $\lambda_\circ$ is the wavelength of a particular two level transition we wish to suppress, spontaneous decay does not occur. This is because a Fourier component of A_μ^{self} with frequency less than $\omega_\circ$ can not meet the parallel mirror boundary conditions and hence it is no longer available in the radiation reaction field surrounding the atom to trigger spontaneous decay. The qualitative structure of the graph is in good agreement with a similar experimental graph,[41] even up to the prediction of an enhancement factor of $3A^{\circ}/2$ when $L = \lambda_{\circ}/2$. This enhanced spontaneous emission rate can be viewed as a cooperative Dicke superradiance phenomenon between the atom and its images.

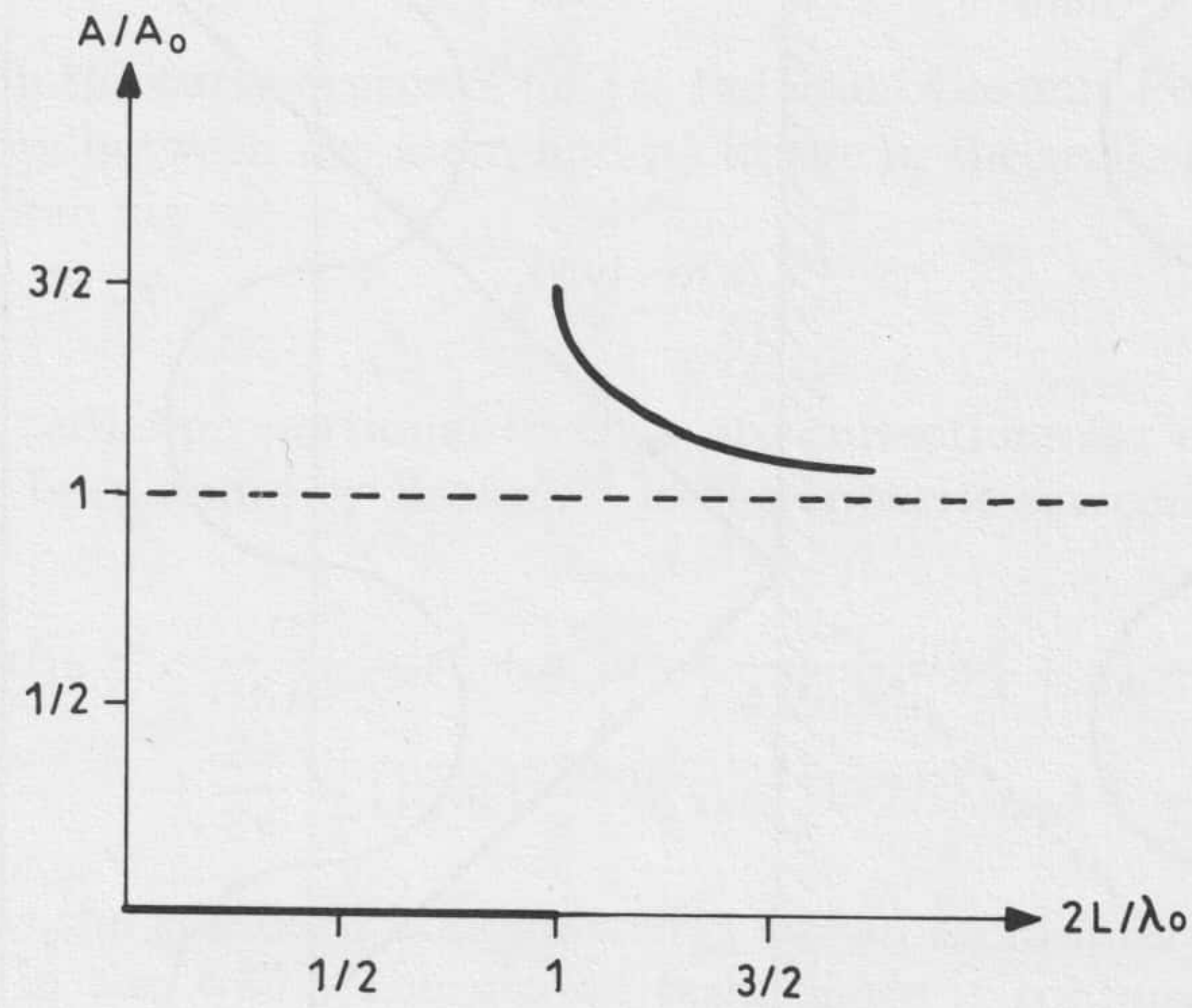

Fig. 2. The Einstein A coefficient between parallel mirrors as a function of the plate spacing L. The freespace value is $A_{\circ}$ and $\lambda_{\circ}$ is the wavelength of the emitted photon. Below the critical plate separation $L = \lambda_{\circ}/2$ spontaneous emission is suppressed. At the criticla separation, the rate is enhanced by a factor of 3/2, but then approaches asymptotically the freespace value as $L \to \infty$.

V. LAMB SHIFT NEAR A SINGLE PLATE

Just as the imaginary energy shift found in equation (13) changes with changing boundary conditions—so does the real part of this shift. Hence one would expect a boundary induced change in the Lamb shift as well as the spontaneous emission rate. Here we give the results of a sample calculation for the apparatus correction to the energy level n of a hydrogen atom which is located a distance R from a single conducting plane. Only one image is needed to construct the necessary Green's function here, in contrast to the double mirror case.

Schematically, the Green's function construction can be viewed as the sum of two Green's functions, where the image Green's function is evaluated at a retarded

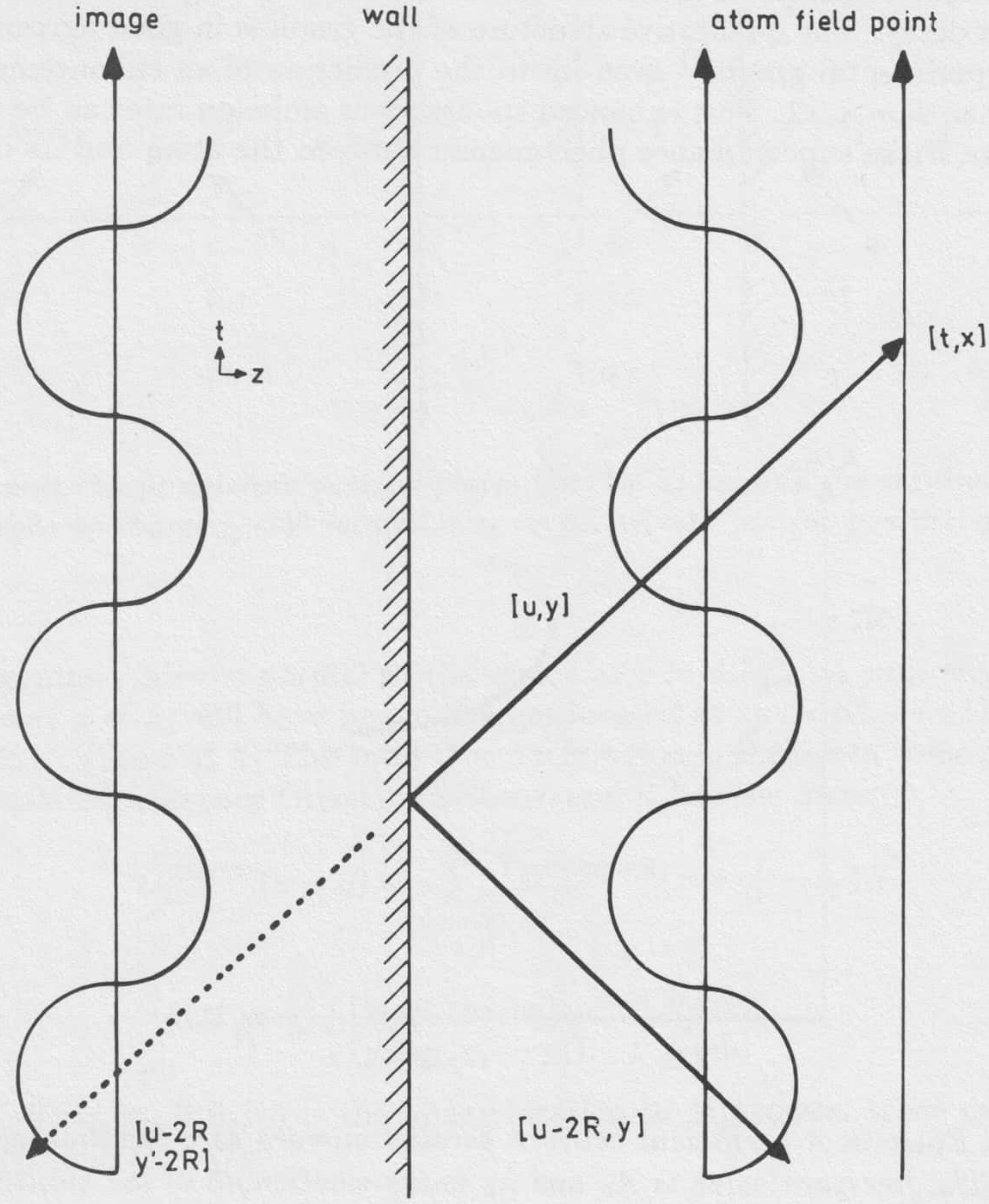

Fig. 3. The value of the electromagnetic Green's function at the point $(t, \mathbf{x})$ *is determined by the current at the source point at* $(u, \mathbf{y})$ *and by the image current at the image source point* $(u - 2R, \mathbf{y}' - 2\mathbf{R}$, *where we define* $\mathbf{y}' := (y_1, y_2, -y_3)$ *and* $\mathbf{R} := (0, 0, R)$. *Notice that the image source point is retarded in both space and time.*

spacetime point. (See Fig. 3.) The energy shift turns out to be given by[31]

$$\Delta E_n = \frac{\alpha}{\pi} \sum_m \omega_{nm}^3 |\mathbf{r}_{nm}|^2 \Big\{ (1 - \zeta_{nm}) \Big[\frac{1}{\mu_{nm}^2} - \frac{f(\mu_{nm})}{\mu_{nm}} + \pi \Theta_{nm} \frac{\cos\mu_{\rm nm}}{\mu_{\rm nm}} \Big] - (1 + \zeta_{nm}) \Big[\frac{g(\mu_{nm})}{\mu_{nm}^2} + \frac{f(\mu_{nm})}{\mu_{nm}^3} + \pi \Theta_{nm} \Big(\frac{\sin\mu_{\rm nm}}{\mu_{\rm nm}^2} + \frac{\cos\mu_{\rm nm}}{\mu_{\rm nm}^3} \Big) \Big] \Big\} \quad (16)$$

where ζ_{nm} is defined as before, and $\mu_{nm} := 2\omega_{nm} R$. We have also defined a step function as $\Theta_{nm} := \Theta(n - m)$ which turns on only if the level n is an excited state.

The functions f and g are defined as

$$f(z) := \int_0^\infty \frac{\mathrm{sint}}{\mathrm{t+z}} dt \qquad \text{and} \qquad g(z) := \int_0^\infty \frac{\mathrm{cost}}{\mathrm{t+z}} dt$$

The expression (16) has two limiting cases of interest. For an atom far from the plate we have ($R \to 0$)

$$\Delta E_n \sim \frac{\alpha}{\pi} \sum_{\substack{m \\ m\neq n}} \omega_{nm}^3 |\mathbf{r}_{nm}|^2 \Bigg\{ \frac{4}{\mu_{nm}^4} + \pi \Theta_{nm} \times \left[(1-\zeta_{nm}) \frac{\cos\mu_{\mathrm{nm}}}{\mu_{\mathrm{nm}}} - (1+\zeta_{nm}) \left(\frac{\sin\mu_{\mathrm{nm}}}{\mu_{\mathrm{nm}}^2} + \frac{\cos\mu_{\mathrm{nm}}}{\mu_{\mathrm{nm}}^3} \right) \right] \Bigg\} \qquad (17)$$

The first term in the curly braces of (17) is the usual Casimir-Polder, long range, Van der Waals energy between the atom and its image in the wall. From this energy one obtains the a force via

$$F = -\frac{\partial(\Delta E)}{\partial R}$$

The additional terms proportional to Θ_{nm} are corrections for excited states. These terms have also been found by Barton.[48] In the opposite extreme of the limit $R \to \infty$ we have

$$\Delta E_n \sim -\frac{\alpha}{16R^2} \langle n|r^2 + z^2|n\rangle + \frac{\alpha}{4Rm^2} \langle n|p_x^2 + p_y^2 - 2p_z^2|n\rangle + \frac{2\alpha}{3\pi} \sum_m \left(|\mathbf{r}_{nm}|^2 - 2|\zeta_{nm}|^2 \right) \ln 2R|\omega_{nm}| + \frac{\alpha}{4\pi m R^2} \qquad (18)$$

The first term is the standard London energy for an atom interacting nonretardedly with its image in the wall. The second term, since it contains components of p^2, corresponds to an anisotropic change in the electron mass. In general, the mass of an electron in a cavity is a tensor quantity; the inertial response of an electron to a given force will depend on the direction of the force. (One way to visualize this is to consider that the electromagnetic part of the electron's mass is bound up in the electric field lines, and that some of these lines are now cut short by the conductor, hence changing the mass.) The third term in equation (18) is something like an anisotropic contribution to the Lamb shift, and the final term is a level independent contribution to the Van der Waals energy. These results also agree with those of Barton,[48] and the reader is refered thither for more detailed discussion of these terms.

VI. $g-2$ OF AN ELECTRON NEAR A MIRROR

In 1989, Hans Dehmelt shared the Nobel Prize in physics for his work with the Penning trap. His group has made to-date the most accurate measurements of the electron g factor, by comparing the cyclotron and spin precession frequencies, ω_{cyc} and ω_{spin} respectively, of an electron electromagnetically bond in such a trap. The electron anomaly factor a can be written as

$$a := \frac{g-2}{2} \equiv \frac{\frac{g}{2}-1}{1} \equiv \frac{\omega_{\mathrm{spin}} - \omega_{\mathrm{cyc}}}{\omega_{\mathrm{cyc}}} \qquad (19)$$

so long as one is in freespace. Dehmelt's result is[49]

$$a_{\mathrm{exp}} = 1\ 159\ 652\ 193\ (4) \times 10^{-12} \qquad (20)$$

Unfortunately the experiment is not carried out in freespace, but rather inside a conducting cavity whose dimension is of the order of 1cm in diameter. Since a_{exp} is compared to a theoretical calculation of $a_{\rm thy}$ using *freespace* Feynman integrals—it becomes of crucial importance to know at what stage of accuracy does the apparatus introduce a systematic error into $a_{\rm exp}$. The trap alters the radiation reaction field, in the self-field picture, and so we would expect both $\omega_{\rm spin}$ and $\omega_{\rm cyc}$ to change, altering the value of $a_{\rm exp}$. There has been quite a confusion in the literature over whether or not there was a first order correction to $\omega_{\rm spin}$ as well as $\omega_{\rm cyc}$.[32,45,49—58] The consensus seems to now be that there is a correction to $\omega_{\rm cyc}$ but no first order boundary induced change in $\omega_{\rm spin}$. This fact is due to a subtle cancellation between apparatus corrections to the electron magnetic moment and to those of the mass.[32,58] To see how this occurs, let us take the simple case of an electron undergoing a cyclotron orbit in a plane parallel to, and a distance R from, a plane conducting mirror. Instead of the Schrödinger action of equation (6) we now use the Pauli action of (7) so that we may include the electron spin in an elementary fashion. With the same approximations as we made in the Schrödinger action to arrive at the action density (9), we now obtain from the Pauli action density (7) the simplified density

$$\begin{aligned} w =& \phi^* \Big[-\frac{1}{2m}\nabla^2 - i\frac{\partial}{\partial t} + \frac{ie}{m}\mathbf{A}^{\rm ext}\cdot\nabla - \frac{e}{2m}\sigma\cdot\mathbf{B}^{\rm ext} \\ &+ \frac{e}{2}A_{\rm o}^{\rm self} + \frac{ie}{2m}\mathbf{A}^{\rm self}\cdot\nabla - \frac{e}{4m}\sigma\cdot\mathbf{B}^{\rm self}\Big]\phi \end{aligned} \tag{21}$$

where we have now set $A_{\rm o}^{\rm ext} \equiv 0$ but $\mathbf{A}^{\rm ext} = (\mathbf{B}^{\rm ext}\times\mathbf{r})/2$. Without the terms labeled with the superscript *self*, equation (21) would be the usual semiclassical expression for an electron with spin, executing Landau orbits in a homogeneous magnetic field. The $A_{\rm o}^{\rm self}$ and $\mathbf{A}^{\rm self}\cdot\nabla$ terms, similar as to before, give rise to mass renormalization, Lamb shift and spontaneous emission effects, while that proportional to $\sigma\cdot\mathbf{B}^{\rm self}$ is responsible for the nonzero value of $g-2$ in freespace.[8,28] Apparatus dependent shifts in the $\sigma\cdot\mathbf{B}^{\rm self}$ energy would alter $\omega_{\rm spin}$ and hence $g-2$. Changes in the cyclotron frequency come about through changes in the $\mathbf{A}^{\rm self}\cdot\nabla$ term.

The boundary condition that $\mathbf{E}_{||}$ and $\mathbf{B}_{\perp}$ vanish identically on the surface of a conductor S can be covariantly written as[56—58]

$$F_{\mu\nu}\, n_\alpha\, \epsilon^{\mu\nu\alpha\beta}|_S \equiv 0, \qquad \forall\beta \tag{22}$$

where $n_\alpha := (0, \hat{\mathbf{n}})$ and $\hat{\mathbf{n}}$ is everywhere normal to the surface S in the restframe of the conductor. This constraint (22) can be met by the axial gauge condition

$$n_\mu\, A^\mu|_S \equiv 0 \tag{23}$$

which gives rise to the Green's function:

$$D_{\mu\nu}(x-y) = -\frac{1}{(2\pi)^4}\int dk\, \frac{e^{ik\cdot(x-y)}}{k^2+i\epsilon}\left(g_{\mu\nu} - \frac{n_\mu\, k_\nu + n_\nu\, k_\mu}{n\cdot k} + \frac{n^2\, k_\mu\, k_\nu}{(n\cdot k)^2}\right) \tag{24}$$

Of course the choice of the Green's function can not affect our answer, since the theory is gauge invariant, but it can greatly simplify our calculations—which is why we use this special Green's function here. Restricting ourselves to the case where we have a single plane surface, we can set $\hat{\mathbf{n}} := \hat{\mathbf{z}}$ and write equation (4) with the help of (24) as

$$\mathbf{A}^{\rm self}(x) = -\frac{e}{(2\pi)^4}\int\!\!\int dy\, dk\, \frac{e^{ik\cdot(x-y)}}{k^2+i\epsilon}\left[\mathbf{j}(\mathbf{y}) - \frac{(\mathbf{k}\cdot\mathbf{j})\hat{\mathbf{z}} + (\hat{\mathbf{z}}\cdot\mathbf{j})\mathbf{k}}{\hat{\mathbf{z}}\cdot\mathbf{k}} + \frac{(\mathbf{k}\cdot\mathbf{j})\mathbf{k}}{(\hat{\mathbf{z}}\cdot\mathbf{k})^2}\right] \tag{25a}$$

$$A_{\rm o}^{\rm self}(x) = -\frac{e}{(2\pi)^4}\int\!\!\int dy\, dk\, \frac{e^{ik\cdot(x-y)}}{k^2+i\epsilon}\left[1 - \frac{\omega^2}{(\hat{\mathbf{z}}\cdot\mathbf{k})^2}\right]\rho(y) \tag{25b}$$

where ρ and $\mathbf{j}$ are the space and time components of the Pauli current j_μ, which are given in (7b). Using the fact that $\mathbf{B}^{\text{self}} = \nabla \times \mathbf{A}^{\text{self}}$, we may now compute the boundary induced effect on the $\sigma \cdot \mathbf{B}^{\text{self}}$ energy term using the method of images as before. After isolating the the electron's magnetic moment μ we can extract the plate correction $\Delta\mu$ as

$$\frac{\Delta\mu}{\mu} = -\frac{\alpha}{4Rm} \tag{26}$$

However, there is a plate induced mass correction similar to the tensorial mass change we saw before in the apparatus dependent Lamb shift expression (18). So long as the electron is constrained to move in a plane parallel to the conducting mirror, this mass correction is given by

$$\frac{\Delta m}{m} = -\frac{\alpha}{4Rm} \tag{27}$$

(Since translation invariance is broken in the $\hat{\mathbf{z}}$ direction, momentum of the electron is not conserved in this direction—which is part of the reason we restrict the motion to a plane parallel to the conductor. The breaking of translation invariance in the z direction also manifests itself in the tensorial character of the electron mass.) The total correction to the magnetic spin energy is, to first order in the unitless parameter α/Rm, given by

$$\begin{aligned} \Delta E^{\text{spin}} &= -\frac{e}{2m}\sigma \cdot \mathbf{B}^{\text{ext}}\left(1 + \frac{\Delta\mu}{\mu}\right) \\ &= -\frac{e}{2m_\circ}\sigma \cdot \mathbf{B}^{\text{ext}}\left(1 - \frac{\Delta m}{m} + \frac{\Delta\mu}{\mu}\right) \\ &= -\frac{e}{2m_\circ}\sigma \cdot \mathbf{B}^{\text{ext}} \end{aligned} \tag{28}$$

where the first order correction vanishes if we express the mass m near the plate in terms of the observed freespace mass $m_\circ$. Hence there is not first order change of the spin precession frequency ω_{spin}. One can now calculate the cyclotron frequency shift. This turns out to be essentially a classical effect, and it *does* remain intact at order α/Rm, and hence is the dominant effect:

$$\omega_{\text{cyc}} \sim \frac{\alpha}{Rm} \tag{29}$$

For the presently used cavities, the systematic error introduced by the cavity wall is about 1 part in 10^{12}, which is precisely the accuracy of the current $g-2$ experiments. It would seem that apparatus contributions to ω_{cyc} are now a limiting factor on the accuracy of this type of measurement of $g-2$.

VII. THE UNRUH EFFECT AND HAWKING RADIATION

We now move along to a more arcane boundary condition in QED which gives rise to the Unruh and the related Hawking radiation. Consider a general quantum mechanical detector. For our purposes, we demand that the detector have a complete set of energy levels, and that it couple to the electromagnetic field. (A hydrogen atom would serve quite well.) At a temperature of $T = 0°K$ an inertially moving detector sees no photons—although some might argue that the detector 'sees' the vacuum fluctuations since they trigger such effects as the freespace spontaneous emission and Lamb shift. From the self-field point of view, however, such radiative effects are an integral part of a complete Dirac equation, say, which incorporates the radiation reaction field. So from our point of view an inertially moving detector registers no photons in a vacuum.

Shortly after Hawking showed that the event horizon of a black hole appears to emit thermal radiation,[59] Unruh proved that a uniformly accelerating detector in a vacuum appears to register a thermal bath of photons at a temperature proportional to the acceleration.[60] It turns out that the two phenomena are related via a conformal transformation. The Unruh radiation can be thought of as being emitted from the event horizon of Rindler space. (Rindler space is a coordinate system used to describe a uniformly accelerating reference frame.) In particular, a detector accelerating uniformly through a vacuum with a constant acceleration a responds *as if* it is immersed in a Planckian distribution of photons at a temperature T given by

$$T = \frac{\hbar a}{2\pi k c} \tag{30}$$

where k is Boltzmann's constant.

This result leads to a rather paradoxical conclusion: An inertial detector near an accelerating detector will see nothing, while the other sees a thermal flux of photons. If the photons are really real and exist as physical entities in the surrounding space—then how come the inertial detector can't see them? To understand this, let's consider the quantum field theoretic, plane wave mode decomposition of the electromagnetic potential, namely

$$A_\mu(x) = \frac{1}{(2\pi)} \int dk \left[a_\mu(k)\, e^{-ik\cdot\mathbf{x}} + \text{h.c.}\right] \tag{31}$$

This decomposition is unique *only* in Minkowski space—hence *only* in Minkowski space is 'the vacuum' uniquely defined. In curved spacetime this decomposition is not unique and in general different observers will see different vacua. Considerations such as these have lead P. C. W. Davies to conclude that the notion of particle—'photon' in this case—breaks down in curved spacetime.[14] From the self-field point of view, the problem is not with the notion of 'particle', but rather with the quantum field notion of 'vacuum'. Here is just one more example of how the idea of a dynamic, fluctuating zeropoint field leads to extreme difficulties and apparent contradictions in other areas of physics.

The Unruh effect can also be calculated from the self-field point of view, but now the interpretation is entirely different. The detector, by the very fact that it is assumed to couple to the EM field, must contain a self-field of its own, given by equation (4). This self-field becomes modified by an accelerating agent in such a way so that it acts back on the detector and drives it into a superposition of excited states. When thermodynamically analyzed, an accelerating detector appears *as if* it is subjected to a bath of thermal photons. But this is just an illusion. The radiation reaction field of the detector is merely being perturbed by the acceleration. There are no real photons surrounding the detector, and hence the concept of 'photon' can be saved, since there is nothing for a neighboring inertial detector to detect. There is no paradox. In general relativity, as Davies has repeatedly emphasized, one may not discuss 'the vacuum' independent of the worldline of the detector which is being used to observe deviations away from this vacuum. This point highlights why differing detectors apparently see different vacua. In self-field theory all the vacua are identical and empty; the differing detectors are only responding to *themselves* in differing fashions.

To carry out the calculation of the Uhruh effect[60] we use the fully covariant version of the self-field theory embodied in the Dirac action of equation (8). The self-field contribution to the total action is then given exactly by equation (10). We use the usual QED causal Green's function $D_{\mu\nu} := -\eta_{\mu\nu} D$ in the Feynman gauge, where

$D(x-y)$ can be written in any of the equivalent forms

$$\begin{aligned} D(x-y) &= \frac{i}{4\pi^2}\left\{\frac{1}{(x-y)^2} - i\pi\delta\big[(x-y)^2\big]\right\} \\ &= \frac{i}{4\pi^2}\,\frac{1}{(x-y)^2+i\epsilon} \\ &= \frac{1}{(2\pi)^4}\int \frac{e^{-ik\cdot(\mathbf{x}-\mathbf{y})}}{k^2+i\epsilon} \end{aligned} \tag{32}$$

We wish now to boost this Green's function into the accelerating frame of Rindler coordinates. If we take the acceleration vector $\mathbf{a} := (0,0,a)$ in the z direction, then in the zt plane, these coordinates can be written as:

$$x_\circ = \sinh(\tau) \qquad y_\circ = \sinh(\tau') \tag{33a}$$

$$x_3 = \cosh(\tau) \qquad y_3 = \cosh(\tau') \tag{33b}$$

with all other components zero. Here τ and τ' are the propertimes associated with the source point x_μ and the field point y_μ, respectively, of the Green's function $D(x_\mu - y_\mu)$ which is comoving with the detector. We have set $\hbar = c = a = 1$. In these coordinates the Green's function (32) becomes, in the dipole approximation where $\mathbf{x} \approx \mathbf{y}$,

$$\begin{aligned} D(x-y) &\approx \frac{1}{16\pi^2}\operatorname{csch}\Big(\frac{i\Delta\tau}{2}+i\epsilon\Big) \\ &= \frac{1}{4\pi^2}\sum_{p=-\infty}^{\infty}\Big(\Delta\tau + 2\pi i p + i\epsilon\Big) \end{aligned} \tag{34}$$

where $\Delta\tau := \tau - \tau'$, and we have expanded the hyperbolic cosecant in an infinite partial fraction expansion. If one now makes the Fourier expansion

$$\psi(x) = \sum_n \psi_n(\mathbf{x})\, e^{-iE_n t} \tag{35}$$

for each ψ appearing in equation (10), one arrives at an expression for the self-field contribution to the total action, given by

$$W^{\text{self}} = -\frac{\alpha}{2\pi}\sum_{n,m}\langle n|\gamma^\mu|m\rangle\langle m|\gamma_\mu|n\rangle \sum_{p=-\infty}^{\infty}\int\!\!\int d\tau\, d\tau' \frac{e^{i\omega_{nm}\Delta\tau}}{\big(\Delta\tau + 2\pi i p\big)^2} \tag{36}$$

This may be converted to a transition probability G per unit time by replacing the double integral with a single integral via the prescription $\int\!\int d\tau d\tau' \to \int d(\Delta\tau)$. Carrying out the integration, and then summing a remaining geometric series yields

$$G = \frac{\alpha}{2\pi}\sum_{\substack{m \\ m<n}}\left[\frac{\hbar\omega_{nm}}{2} + \frac{\hbar\omega_{nm}}{\exp\big(\frac{2\pi c\omega_{nm}}{a}\big)-1}\right] \tag{37}$$

From the form of this equation, we see that by taking $a \to 0$ we recover only the term $\hbar\omega_{nm}/2$ in the curly braces. This is suggestive of the zeropoint field of standard QED, but here we have only the radiation reaction field of an inertial detector. This observation supports the suggestion of Jaynes, Milonni and others that the zeropoint fluctuations are perhaps a mathematical subterfuge which mimic the physical radiation reaction field.[12,13] When $a > 0$ we see that there is a Planckian contribution to the transition rate, which corresponds precisely to the Unruh temperature given in equation (30). But there are no real thermal photons—just as in the inertial case the detector is merely responding to itself. Only now this self-response has been modified by the force required to maintain the detector in an accelerating frame. As we mentioned earlier, a detector outside of a black hole sees a similar thermal radiation. Indeed, a detector accelerating at a rate a is by the equivalence principle indistinguishable from a detector at rest in a gravitational field $\mathbf{g}$ of strength $g = a$. From the self-field view, then, Hawking radiation is in some sense just as fictitious as the Unruh radiation. It would be our contention that the black hole is not emitting radiation in the usual sense, but rather it is perturbing the metric around itself such that a nearby detector responds to the curvature as if it were bathed with thermal photons.

VIII. CONCLUSION

We have summarized here how the self-field theory of QED can be used to account satisfactorily, at least to order α, for an array of boundary induced changes in the radiative corrections found in QED. We have emphasized that it is not necessary to invoke the notion of zeropoint fluctuations to construct an interpretive framework of a boundary modified radiative effects. One can view the whole process for the point of view of a quantum analog to classical radiation reaction theory. This is consistent with the self-field point of view, which looks upon all radiative effects as arising from the correct inclusion of the back reaction upon a charge of its own self field. It is our position that an electromagnetic field does not exist independent of the source which produced it—in the spirit of the quote from Einstein which appeared at the beginning of this work.

ACKNOWLEDGEMENT

I would like to thank Professor A. O. Barut for all of his help and guidance in the execution of the calculations upon which this this work has been based, and for all the work he did in organizing such a great conference in İstanbul. I would also like to thank G. Barton and M. Kreuzer for some interesting discussions and correspondences concerning cavity QED.

REFERENCES

1. P. A. M. Dirac, Proc. R. Soc. London, Ser. **A**, 243 (1927).
2. *Selected Papers on Quantum Electrodynamics*, edited by J. Schwinger (Dover, New York, 1958).
3. E. A. Power, *Introductory Quantum Electrodyamics* (Longmans, London, 1964).
4. S. Weinberg, Rev. Mod. Phys **61**, 1–24 (1989).
5. T. A. Welton, Phys. Rev. **74**, 1157 (1948).
6. H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).
7. H. Grotch and E. Kazes, Phys. Rev. **D 13**, 2851 (1976).
8. A. O. Barut, J. P. Dowling, and J. F. van Huele, Phys. Rev. **A 38**, 4405 (1988).
9. H. G. B. Casimir, Proc. K Ned. Akad. Wet. **51**, 793 (1948).
10. H. G. B. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).
11. G. Barton and N. S. J. Fawcett, Phys. Rep. **170**, 1–95 (1988).
12. E. T. Jaynes, "Probability in Quantum Theory," to appear in *Proceedings of the 1989 Workshop on Complexity, Entropy, and the Physics of Information* (Addison-Wesley Publishing Co.).
13. P. W. Milonni, Phys. Script. **T 21**, 1020 (1988).

14. N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, U.K., 1982).

15. E. Schrödinger, Ann. Phys. **82**, 265 (1926).

16. E. Fermi, Rend. Lincei **5**, 795 (1927).

17. M. D. Crisp and E. T. Jaynes, Phys. Rev. **179**, 1253 (1969).

18. A. O. Barut and J. P. Dowling, "Self-field QED: The two level atom," (submitted to Phys. Rev. A).

19. P. A. M. Dirac, Proc. R. Soc. London, Ser. **167 A**, 148 (1938).

20. A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (Dover, New York, 1980), Chapter VI.

21. G. Süssman, Seit. f. Phys. **131**, 629 (1952).

22. J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys **17**, 157 (1945).

23. A. O. Barut and J. Kraus, Found. Phys. **13**, 189 (1983).

24. A. O. Barutand J. Kraus, Phys. Rev. **D 16**, 161 (1977).

25. A. O. Barut and J. F. van Huele, Phys. Rev. **A 32**, 3187 (1985)

26. A. O. Barut and Y. I. Salamin, Phys. Rev. **A 37**, 2284 (1988).

27. A. O. Barut and Y. I. Salimin (submitted to J. Math. Phys.).

28. A. O. Barut and J. P. Dowling, Z. Naturforch. **44**a, (November, 1989).

29. A. O. Barut and N. Unal (unpublished).

30. A. O. Barut and J. P. Dowling, Phys. Rev. **A 36**, 649 (1987).

31. A. O. Barut and J. P. Dowling, Phys. Rev. **A 36**, 2550 (1987).

32. A. O. Barut and J. P. Dowling, Phys. Rev. **A 39**, 2796 (1989).

33. A. O. Barut and J. P. Dowling, "QED based on self-fields: On the origin of the thermal radiation detected by an accelerating observer," (submitted to Phys. Rev. A).

34. P. W. Milonni, Phys. Rev. **A 25**, 1315 (1982).

35. J. R. Ackerhalt, P. L. Knight, and J. H. Eberly, Phys. Rev. Lett. **30**, 456 (1973).

36. J. R. Senitzky, Phys. Rev. Lett. **31**, 955 (1973).

37. P. W. Milonni, J. R. Ackerhalt, and W. A. Smith, Phys. Rev. Lett. **31**, 958 (1975).

38. P. W. Milonni and W. A. Smith, Phys. Rev. **A 11**, 814 (1975).

39. J. Dalibard, J. Dupont-Roc, and C. Cohen-Tannoudji, *J. Phys.* (Paris) **43**, 1617(1982).

40. P. Goy, J. M. Raymund, M. Goss and S. Haroche, Phys. Rev. Lett. **50**, 1903 (1983).

41. R. G. Hulet,E. S. Hilfer, and D. Kleppner, Phys. Rev. Lett. **55**, 2137 (1985).

42. A. O. Barut, Found. Phys. **17**, 549 (1987).

43. A. O. Barut, Found. Phys. **18**, 95 (1987).

44. A. O. Barut, Ann. Phys. **45**, 31 (1988).

45. G. Barton, Proc. R. Soc. London, Ser. **320 A**, 251 (1970).

46. P. W. Milonni and P. L. Knight, Opt. Commun. **9**, 119 (1973).

47. M. R. Philpott, Chem. Phys. **53**, 101 (1976).

48. G. Barton, J. Phys. **7 B**, 2134 (1974).

49. H. Dehmelt, Phys. Rev. **D 34**, 722 (1986).

50. M. Babiker and G. Barton, Proc. R. Soc. London, Ser. **326 A**, 255,277 (1972).

51. G. Barton and H. Grotch, J. Phys. **10 A**, 1201 (1977).

52. E. Fischbach and N. Nakagawa, Phys. Rev. **D 30**, 2356 (1984).

53. L. S. Brown, G. Gabrielse, K. Helmerson, and J. Tan, Phys. Rev. **A 32**, 3204 (1985).

54. L. S. Brown, G. Gabrielse, K. Helmerson, and J. Tan, Phys. Rev. Lett. **55**, 44 (1985).

55. D. G. Boulware, L. S. Brown, T. Lee, Phys. Rev. **D 32**, 729 (1985).

56. K. Svozil, Phys. Rev. Lett. **54**, 742 (1985).

57. M. Kreuzer and K. Svozil, Phys. Rev. **D 34**, 1429 (1986).

58. M. Kreuzer, J. Phys. **21 A**, 3285 (1988).

59. S. Hawking, Commun. Math. Phys. **43**, 199 (1975).

60. W. G. Unruh, Phys. Rev. **D 14**, 870 (1976).