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NONLINEAR NONLOCAL CLASSICAL FIELD THEORY OF QUANTUM PHENOMENA

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Abstract—It is generally thought that there is a mysterious dividing line between classical physics and quantum physics: one objective, deterministic and continuous; the other discrete, indeterminate and subjective even at the single entity level. Such a division seems to be provisional and unsatisfactory in natural philosophy, and physicists never stopped to search for a deterministic and objective substratum of quantum phenomena. There is a recent revival of thought that matter and light and their interactions can be described by the ideas of classical field theories of continuous media and yet all the quantum effects can be reproduced, when the Dirac field for the electron is treated as a classical complex field. The discreteness in quantum physics is on the same footing as the discrete frequencies of a membrane due to the boundary conditions, and this discreteness disappears when radiative effects are included. The nonlinear nature of coupled matter and field equations plays a crucial role. Furthermore, the particle mechanics is a limit of a deterministic quantum theory of single events which is distinct from the quantum theory of repeated events for which we use probabilistic concepts.

1. MAXWELL–DIRAC CLASSICAL FIELD THEORY

(A) The action principle

The interactions of electrons with electromagnetic radiation is governed by the coupled Maxwell–Dirac equations. These follow from an action principle with an action density (Lagrangian)

$$\mathscr{L} = \sum_k \{\bar{\psi}_k(\gamma_\mu i\,\partial^\mu - m)\psi_k - e_k\bar{\psi}_k\gamma^\mu A_\mu\psi_k\} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{1}$$

We have introduced distinct fields ψ_k for a number of particles interacting via the electromagnetic potential A_μ with $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$.

It is both easier and more fundamental to describe the matter by Dirac fields rather than Klein–Gordon or Schrödinger fields, because the action is then linear in ∂_μ, and also spin 1/2-fields seem to represent correctly the absolutely stable fundamental entities of matter, like the electron and neutrino. All other particles observed in nature seem to be composites.

(B) Combining relativity with quantum theory

We shall treat the fields A_μ and ψ as entirely classical fields in Minkowski space and shall obtain physical quantities directly from the action without the need of neither quantized fields, nor the probability interpretation of these fields. This immediately removes one of the major obstacles in quantum field theory that of reconciling special relativity with the Hilbert space formulation of probabilistic quantum theory. The former is a four-dimensional framework: The Hilbert space is the space of functions at fixed t in $L^2(R^3)$ and has to be reconstructed anew in every frame [1]. The difficulty has been overcome so far in perturbation theory where one uses in every step free solutions of the Dirac equation for which the probability density is constant everywhere so it does not matter how the time-slice in the Minkowski space is taken. But not in a general interacting theory. On the other hand in a classical relativistic field theory there is no problem with relativity for fields, nor for extended elementary objects. The Planck's constant $\hbar$ comes in only if in ψ-field we wish to express frequency ω and wave vector $\mathbf{k}$ in terms of the energy E and momentum $\mathbf{p}$, respectively. But we can work also entirely in terms of frequencies and wave vectors.

(C) The concept of self field

From the action principle (1) we derive, as usual, the coupled Maxwell and Dirac equations:

$$(\gamma^\mu i\,\partial_\mu - m_k)\psi_k = e_k\gamma^\mu\psi_\mu(x)A_\mu \tag{2}$$

$$F^{\mu\nu}_{,\nu} = -\sum_k e_k\bar{\psi}_k\gamma^\mu\psi_k \equiv \sum_k j^\mu_k \equiv j^\mu \tag{3}$$

Choosing a convenient gauge $A^\mu_{,\mu} = 0$, we can solve equation (3) in terms of the Green's function with appropriate initial and boundary conditions

$$A_\mu(x) = \int \mathrm{d}y D(x-y)j^\mu(y) + A^0_\mu(x) \tag{4}$$

where $A^0_\mu(x)$ is some free field (a field whose sources are far away), which we shall put equal to zero in this work. The field (4) when inserted into (2) gives to the kth particle ψ_k a potential due to all other particles, as well as a potential produced by its own current. The latter which is not included in ordinary quantum mechanics and quantum electrodynamics is responsible for all the so-called radiative processes, and is the essential feature of the new approach.

We can thus eliminate in (1) the A_μ-field using (4) and obtain the action

$$\mathcal{W} = \int \mathrm{d}x \sum_k \left\{\bar{\psi}_k(\gamma^\mu i\,\partial_\mu - m_\mu)\psi_k - \sum_l \frac{1}{2}\int \mathrm{d}y j^k_\mu(x)D(x-y)j^\mu_l(y)\right\} \tag{5}$$

This is the nonlinear nonlocal action referred to in the title. The self-energy action comes from the term $l = k$. This is a simple looking current–current interaction. But it is remarkable that the whole-body of quantumelectrodynamics in free space, in media, in cavities, in accelerating frames, . . . follows directly from this action in a unified manner, when the appropriate currents and appropriate Green's functions are inserted.

(D) Comparison with perturbative quantumelectrodynamics (QED)

In standard QED, the self-energy terms in (5) are simply dropped. Instead a separate quantized radiation field A^{rad} is introduced with its independent degrees of freedom. One second-quantizes both the free fields ψ and A^{rad} (as a sum of oscillators) and then these free fields are coupled perturbatively in the usual way [3].

According to equation (4), however, every field A_μ has a source. If there is no current there is also no field. The electromagnetic field has no independent degrees of freedom; it is fully determined by the nature of the source. If the source has quantized properties (first quantized) then the light that it puts out will also have such quantized properties. It is therefore sufficient to deal only with sources and detectors, as given by the second term in the action (5).

The nonlinear action (5) solved iteratively gives in the lower order of iterations the same results as the low orders of perturbation theory. But the former clearly allows nonperturbative treatment and has a number of fundamental implications. Furthermore, the renormalization procedures are quite different in both theories.

(E) Fundamental implications

Elimination of vacuum fluctuations. In the self-field formulation there are no vacuum fluctuations, nor zero-point energy, both due to the assumption of the existence of a quantized radiation field. In other words vacuum is vacuum—no sources present. The almost infinite zero-point energy of the universe is an embarrassment, and is contrary to experiment, which causes a tremendous problem in gravitational theories. Similarly, the ever present vacuum fluctuations are counter-intuitive, and can be replaced by the properties of the source. Also the so-called vacuum-polarization contributions in the presence of fields, e.g. in the Lamb-shift, can be obtained from the self field, as we shall see.

Reality of the ψ-field. A second fundamental implication of the new approach is that equation (4) attributes to the current $j_\mu = e\bar{\psi}\gamma^\mu\psi$, and hence to the ψ-field, a real material existence, not just a probability amplitude. In the standard interpretation of quantum mechanics, the current of the ψ-field does not produce a field of its own.

Continuous energy spectrum. A third major implication is that now the Schrödinger or the Dirac equation resulting from (1) or (5) carries always its self-field with it:

$$(i\,\partial_t - H_0)\psi_k = e_k^2\gamma^\mu\psi_k\int \mathrm{d}yD(x-y)\bar{\psi}_k\gamma_\mu\psi_k \tag{6}$$

where we have put the effect of all other particles $l \neq k$ into H_0. Only if we neglect the right-hand side, we can get a discrete spectrum of H_0, say the Coulomb problem. With self-energy the system always has continuous spectrum, with a stable ground state. Instead of the discrete spectrum we get spectral concentrations or resonances at certain energy values. This is also what is observed experimentally, so that the discreteness of quantum theory is an idealization. Nor is it necessary to assume discrete photons. In fact experimentally, due to the line widths of the spectral lines, the photons that can be absorbed or emitted do not need to have sharp frequencies.

Many reasons are given why one should quantize the electromagnetic field, such as the photoelectric effect, Compton effect, Planck's distribution law, Lamb-shift, Casimir effect, and so on. But all these effects can and have been calculated on the basis of the sources of the fields [3].

In the next section we indicate the basic steps in the evaluation of the action (5).

(F) Classical and quantumelectrodynamics

The approached based on self-fields is entirely along the lines of classical electrodynamics. The only difference between classical and quantumelectrodynamics is in the form of the current j_μ. In fact equation (5) formally is valid for both. In classical electrodynamics the currents are taken to be those of point relativistic particles

$$j^\mu(x) = \sum_k e_k\int \mathrm{d}\tau\dot{x}_k^\mu(\tau)\delta(x - x_k(\tau)) \tag{7}$$

where τ is an invariant time and $x^\mu(\tau)$ the trajectories, $\dot{x}_k^\mu = \mathrm{d}x_\mu^\mu/\mathrm{d}\tau$. More generally point particles can be replaced by strings or membranes, or in fact also by spinning classical particles.

The only reason that pure classical electrodynamics does not account for experimental atomic observations is that the current (7) does not reflect the wave and the spin properties of the electrons contained in ψ.

(G) Antiparticles

The existence of antiparticles has a natural explanation in terms of Dirac fields. We have to consider, from the factorization of the Klein–Gordon equation, two Dirac equations, not one:

$$\begin{aligned}(\gamma^\mu i\,\partial_\mu - m)\psi_I &= 0\\ (\gamma^\mu i\,\partial_\mu + m)\psi_{II} &= 0,\end{aligned} \tag{8}$$

and take only the positive energy solutions of both of these equations. The negative energy solutions of ψ_I are equivalent to positive energy solutions of ψ_{II}. When coupled to the electromagnetic field, these negative energy solutions represent oppositely charged particles, hence antiparticles.

2. DEVELOPMENT OF THE THEORY AND RESULTS

The further development of the theory [3] proceeds from (5) and (6).

(A) Many-body wave equations

The variation of (5) with respect to the fields ψ_k separately leads to coupled nonlinear wave equations of the Hartree-type. In quantum theory we can also use the method of configuration space: The action (5) can be rewritten in terms of the composite fields

$$\Phi(x_1, x_2 \cdots x_N) = \psi_1(x_1)\psi_2(x_2) \cdots \psi_N(x_N)$$

Then the variation of the action with respect to Φ, which is a weaker extremum principle than the first variation principle, leads to a linear equation for Φ which is the relativistic generalization of the standard many-body Schrödinger equation in configuration space [4]. We give here only the two-body equation

$$\left[(\gamma^\mu i\,\partial_\mu - m_1) \otimes \gamma \cdot n + \gamma \cdot n \otimes (\gamma^\mu i\,\partial_\mu - m_2) + e_1 e_2 \frac{\gamma^\mu \otimes \gamma_\mu}{r_\perp} + V^{\text{Self}}\right]\Phi(x, x_1) = 0$$

This is a 16-component wave equation in the tensor-product space of two 4-component Dirac spinors, n^μ is a time-like four vector related to the choice of a common invariant time parameter τ, and $r_\perp$ the relativistic relative distance perpendicular to n^μ.

This equation has been studied extensively and applied to the hydrogen atom and positronium [5]. It is remarkable that effects calculated term by term by Feynman perturbation graphs can be evaluated in closed form from the solutions of a wave equation.

(B) Understanding the radiative processes

The most interesting and crucial part of quantumelectrodynamics is the understanding of radiative processes attributed generally to vacuum fluctuations of the quantized radiation field. We give here the simple intuitive picture of the most important such effects as spontaneous emission, Lamb-shift and anomalous magnetic moment, that arises from the self-fields of the particles.

Consider the electron in the Coulomb field of the nucleus. In a stationary state the electron does not radiate. But the stationary states are not the exact states of the full equation (6). Corresponding to an iterative solution of this equation, we begin with a state which is a superposition of two stationary states. Now the current of the electron is oscillatory. The electron behaves like an antenna and we can evaluate using equation (4) the field A_μ around the atom. From these we can evaluate the electric and magnetic fields **E** and **B** around the atom. These fields have two parts, a near field going like $1/r^2$ and a far field going the $1/r$. In the far field wave zone we can then calculate the Poynting vector

$$\mathbf{S} = \frac{1}{c}\mathbf{E} \times \mathbf{B}$$

Then integrating the flux over a large sphere around the atom we can evaluate the power of the atom as in antenna which is the rate of spontaneous emission, the inverse of which gives the lifetime [7].

The oscillatory near field acts back on the atom resulting in a self-induced Stark effect or energy-shift which is the Lamb-shift.

The magnetic field produced by the electron itself is proportional to the external field in which the electron moves. Thus the net measured magnetic energy is $\boldsymbol{\mu} \cdot \mathbf{B}_{\text{tot}} = \boldsymbol{\mu} \cdot \mathbf{B}_{\text{ext}}(1 + \lambda)$. The proportionality constant is thus the anomalous magnetic moment. In fact part of the Lamb shift can be attributed to the expectation value of the magnetic self energy $\boldsymbol{\mu} \cdot \mathbf{B}^{\text{Self}} = \lambda\boldsymbol{\mu} \cdot \mathbf{B}^{\text{ext}}$. To first order of iteration one gets the well known result $\lambda = \alpha/2\pi$, the anomalous magnetic moment of the electron [3].

(C) Renormalization

Renormalization in electrodynamics (both classical and quantum) is a procedure to associate a definite mass to a free electron which then moves with constant velocity. Hence the self-energy or radiation reaction of a free electron should be already contained in its mass. The self energy effects calculated in the presence of interactions depends on the parameter Z of external interactions. We must make sure that when $Z \to 0$, the self energy effects must vanish. Thus all remaining terms in this limit $Z \to 0$ must be subtracted. The self energy calculations with free particles diverge. So we evaluate everything in external fields for localized solutions and subtract the terms which remain in the limit $Z \to 0$.

In conclusion, we have tried to show that there is no need of abandoning the scientific method, scientific objectivity, logic and determinism when going from classical physics to quantum physics. The continuity of the great traditions of scientific development can be maintained also in quantum domain. For a deterministic quantum theory of single events we refer to a recent review [6].

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