FOUNDATIONS OF SELF-FIELD QUANTUMELECTRODYNAMICS

A.O. Barut

Department of Physics, University of Colorado
Boulder, Colorado, 80309, USA

The program of selffield approach to Quantumelectrodynamics and radiative processes is almost completed. We give here the main principles of this theory which is conceptually much simpler than the standard perturbative QED. In this formulation the electromagnetic and the electron fields are not quantized; it is a classical field theory. A complete relativistic dynamics of two or more particles interacting via the electromagnetic field is developed by virtue of the new approach. We review numerical results obtained.

I. INTRODUCTION

There are many approaches to radiative processes, or more generally, to electromagnetic interactions of charged particles. We should welcome this multitude because different ways of looking at the same physical phenomena can only bring clarity and hopefully enlightenment. I list those different formulations which are definite and more or less complete:

(i) Second quantized quantum field theory, or the perturbative QED[1].

(ii The S matrix theory of electromagnetic interactions, either from unitarity, analyticity and successive pole approximation[2], or from regularization of the product of distributions[3]. Both of these lead to the renormalized perturbation theory with particles on the mass shell.

(iii) Path integral method. Either path integrals of Maxwell–Dirac fields[4], or path integrals directly from the classical particle trajectories[5].

(iv) Source theory[6].

(v) Selffield quantumelectrodynamics.

Of these only the selffield approach is in the long tradition of classical radiation theory and classical electrodynamics and is the subject of these lectures.

It is often stated that a large number of radiative phenomena conclusively show that the electromagnetic field, and further the electron's field, is quantized as a system of infinitely many oscillators with their zero point energies. The radiative phenomena are listed in Table I. We shall show that all these processes can also be understood and calculated in the selffield approach which does not quantize the fields. The quantum properties of the electromagnetic field are reduced here to the quantum properties of the source. One avoids thereby some of the difficulties of the quantized

New Frontiers in Quantum Electrodynamics and Quantum Optics
Edited by A. O. Barut, Plenum Press, New York, 1990

fields, such as the infinite zero point energy and other infinities of the perturbative QED.

TABLE I. RADIATIVE PROCESSES

Spontaneous emission
Lamb shift
Anomalous magnetic moment
Vacuum polarization
Casimir effect between parallel plates
Casimir Polder potentials
Planck-distribution law for blackbody radiation
Unruh effect
QED in cavities
$e^+ - e^-$ system:
positronium spectrum
positronium annihilation
pair production and annihilation
$e^+ - e^-$ scattering
Relativistic many body problem with retardation
Electron - photon system:
photoelectric effect
Compton effect
Bremsstrahlung

This lecture tells the story of the developments of selffield QED and it is good to begin from the beginning, namely the classical electrodynamics.

II. CLASSICAL ELECTRODYNAMICS

The selfconsistent treatment of coupled matter and electromagnetic field goes back to H.A. Lorentz[7]. The electromagnetic field has as its source all the charged particles which in turn move in this total electromagnetic field. We have thus the Maxwell's equations coupled to the equations for matter:

$$\left.\begin{aligned} &1) \qquad F_{\mu\nu}{}^{,\nu} = -j_\mu \\ &2) \text{ Equation of motion of matter in the field } F \end{aligned}\right\} \tag{1}$$

These equations, both, can be derived from a single action principle. It has the general from

$$W = \int [\text{Kinetic energy of matter } - j_\mu A^\mu - 1/4\, F_{\mu\nu}F^{\mu\nu}] \tag{2}$$

The last term is the action density of the field and the middle term represents the interaction of the matter current with the field.

We shall keep this general framework throughout also for quantum electrodynamics. The only change will be in the specific form of the current or how we describe the matter, the electron.

Classical electrodynamics *per se* is usually associated with the current of point particles moving along wordlines. But we can have more general extended sources of currents, as we shall see. For a number of point particles the current is given by

$$j_\mu(x) = \sum_i e_i \int ds_i \dot{x}_{i\mu}(s_i)\delta\,(x - x_i(s_i)) \tag{3}$$

Hence the fundamental equations are

$$F_{\mu\nu}{}^{,\nu} = -j_\mu = -\sum_i e_i \int ds_i \dot{x}_i(s_i)\delta\,(x - x_i(s_i)) \tag{4}$$

Here s_i are invariant time-parameters on the worldlines of the particles, and dots represent differentiation with respect to these times.

The equations of motions of the worldlines are

$$m_i \ddot{x}_{i\mu} = e_i F_{\mu\nu}\dot{x}_i^\nu \quad , i = 1, 2, 3, \ldots \tag{5}$$

It is essential for the selfconsistency of our system that the field F entering the last equation is the field produced by all the particles including the particle i, namely the selffield. Hence we divide F into two parts

$$m_i \ddot{x}_{i\mu} = e_i F_{\mu\nu}^{(\text{other particles})}\dot{x}_i^\nu + e_i F_{\mu\nu}^{\text{self}}\dot{x}_i^\nu \tag{6}$$

The selffield can be obtained from the Lienard-Wiechert potential

$$A_\mu(x) = \int dx_\mu(s) D\,(x - x(s)) = e \int ds \dot{x}_\mu(s) D\,(x - x(s)) \tag{7}$$

but is formally infinite at the position of the particle. It must be treated properly, for example, by analytic continuation onto the world line[8]. This leads to the final Lorentz-Dirac equation for each particle (in natural units $c = \hbar = 1$)

$$m\ddot{x}_\mu = eF_{\mu\nu}^{\text{ext}}\dot{x}^\nu + \frac{2}{3}\, e^2 \left(\dddot{x}_\mu + (\ddot{x})^2 \dot{x}_\mu\right) \tag{8}$$

This is the basic nonperturbative equation of classical electrodynamics. Here m is now the renormalized mass. Furthermore we must find solutions of this equation which have the property that whenever the external force is zero the electron moves like a free particle, $m\ddot{x}_\mu = 0$, that is the second term must vanish together with the external field. This is part of the renormalization program. The important feature of

this equation is that all radiative effects are now expressed in a closed, we repeat, nonperturbative way. The price we pay for this is that the equation is not only nonlinear but also contains the third derivatives. The selffield approach to quantumelectrodynamics has the goal of finding the analogous nonlinear, nonperturbative equation in the case of quantum currents. It is clear that radiative effects like the Lamb shift, anomalous magnetic moment, spontaneous emission, etc. have their counterparts also in classical electrodynamics.

As a second example of a classical current we consider the classical model of the Dirac electron which describes a spinning and charged relativistic point particle. In this model the worldline of the point particle is a helix, called zitterbewegung, and the orbital angular momentum of the helix in the rest frame of the center of mass accounts for the spin and the magnetic moment of the particle. The generalization of the Lorentz-Dirac equation for this case has recently been given[9]:

$$\dot{\pi}_\mu = eF^{\text{ext}}_{\mu\nu}v^\nu + e^2\left(g_{\mu\nu} - \frac{v_\mu v_\nu}{v^2}\right)\left[\frac{2}{3}\frac{\ddot{v}^\nu}{v^2} - \frac{9}{4}\frac{(v\cdot\dot{v})\dot{v}^\nu}{v^4}\right] \tag{9}$$

where

$$\pi_\mu = p_\mu - eA_\mu,\ v = \dot{x} \text{ and } v^2 \neq 1 \text{ due to spin.}$$

There are other classical models of the electron. A remarkable one is due to Lees[10] amd Dirac[11] in which a charged shell is held stable with a surface tension. In the equilibrium position the surface tension can be expressed in terms of the mass of the electron so that this model has exactly again two parameters, mass and charge, like the point worldline. The Lorentz-Dirac equation for this model to my knowledge has not been worked out yet.

III. SCHRÖDINGER AND DIRAC CURRENTS QUANTUMELECTRODYNAMICS

Quantumelectrodynamics has the same two basic equations (1). Only the form of the current j is different. According to Schrödinger and Dirac the electron is described not by a worldline but by a field $\psi(x,t)$ and the basic coupled equations (1) become

$$F^{,\nu}_{\mu\nu} = -j_\mu \quad , F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$$

and

$$(\gamma^\mu i\partial_\mu - m)\,\psi(x) = e\gamma^\mu\psi(x)A_\mu(x) \tag{10}$$

for the relativistic Dirac case, and

$$i\frac{\partial\psi}{\partial t} = \left(-\frac{1}{2m}\left[(\vec{p} - e\vec{A})^2\right] + eA_0\right)\psi \tag{11}$$

for the nonrelativistic Schrödinger case. The currents for a number of electrons is

$$j_\mu(x) = \sum_i e_i\bar{\psi}_i(x)\overset{(i)}{\gamma}{}_\mu\psi_i(x) \tag{12}$$

with a similar expression for the Schrödinger current.

Again the field A_μ is the sum of an external and a selffield parts:

$$A_\mu = A^{\text{ext}}_\mu + A^{\text{self}}_\mu \tag{13}$$

With the choice of gauge $A^{\mu}_{,\mu} = 0$ the Maxwell equations become

$$\Box A_\mu = j_\mu(x) = \sum_i e_i \bar{\psi}_i(x) \overset{(i)}{\gamma}_\mu \psi_\mu(x) \tag{14}$$

so that the selffield can be expressed in terms of the current as

$$A_\mu(x) = \int dy D(x-y) j_\mu(x) \tag{15}$$

where $D(x-y)$ is the appropriate Green's function corresponding to initial and boundary conditions. Equation (15) is our generalized Lienard-Wiechert potential. Thus the light emitted by a source depends on the nature and preparation of the current, and also on the nature of the environment determining the Green's function. Furthermore the whole light cone where ψ is different from zero contributes to the field at the field point and not just a single intersection of the worldline with the light cone, as in the case of a point particle.

Thus the selffield can be eliminated from the coupled Maxwell-Dirac equations. Inserting A_μ into the equation of motion we obtain

$$\left\{\gamma^\mu \left(i\partial_\mu - e_k A^{\text{ext}}_\mu\right) - m_k\right\} \psi_k(x) = e_k \gamma^\mu \psi_k(x) \int dy D(x-y) \sum_i e_i \bar{\psi}_i(y) \gamma_\mu \psi_i(y) \tag{16}$$

Here A^{ext} is a fixed external field whose sources are far away and not dynamically relevant. In the next Section we shall treat two or many body systems in which we shall eliminate completely all the fields in favor of the currents. Eq. (16) is a nonlinear integral equation for ψ analogous to the nonlinear equation of the classical electrodynamics. The corresponding equation for the Schrödinger case is ($\hbar = 1$)

$$i\frac{\partial \psi}{\partial t} = \left[-\frac{1}{2m}\left(\vec{p} - e\vec{A}^{\text{ext}} - e\vec{A}^{\text{self}}\right)^2 + e\left(A_0^{\text{ext}} + A_0^{\text{self}}\right)\right]\psi \tag{17}$$

where the selfpotentials are

$$A_0^{\text{self}} = \int dy D(x-y) \sum_k e_k \psi_k^*(y)\psi_k(y), \;\; \vec{A}^{\text{self}}(x) = \int dy D(x-y) \sum_k \psi_k^*(y) \frac{\nabla}{i} \psi_k(y) \tag{18}$$

In writing these equations we have assumed that the ψ-current is an actual material charge current, and not just a probability current. Thus we are inevitably led to contemplate the interpretation and foundations of quantum theory. The foundations of quantumelectrodynamics and that of quantum theory must be the same, for quantum mechanics was invented to understand the interactions between light and matter. Not surprisingly, it was Schrödinger who first formulated the selfconsistent coupled Maxwell and matter field equations, i.e., the program of Lorentz, for the new wave mechanics and insisted that for the selfconsistency of the theory the self field of the electron must be included as a nonlinear term. Schrödinger however calculated only the static part of the selfenergy and obtained unacceptable large selfenergies. Subsequently quantum electrodynamics went into a different direction. The selffield was dropped completely. Instead, one introduced a separate quantized radiation field with its own new degrees of freedom and coupled this to the quantized matter field. In the selffield approach the electromagnetic field has no separate degrees of freedom, they are determined by the source's degrees of freedom, but then we must include the full nonlinear selffield term. We shall come to this duality between the two approaches and to the questions of interpretation of quantum theory after the developments of the selffield QED.

IV. RADIATIVE PROCESSES IN AN EXTERNAL (COULOMB) FIELD

The basis of selffield quantumelectrodynamics is conceptually very simple and is completely expressed by the single equation (16). All QED processes in an external field listed in Table I should be derived from this single equation. To perform actual calculations it is much simpler and more direct to work with the action rather than with the equations of motion. The action W can, up to an overall δ-function, be related to the energies of the system for bound state problems, and to the scattering amplitude for scattering problems.

The action for the system (10) is

$$W = \int dx \left[\bar{\psi}(x)(\gamma^\mu i\partial_\mu - m)\psi(x) - e\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right] \tag{19}$$

Here we shall express $A_\mu(x)$ in terms of ψ using (15). For bound state problems the action of the electromagnetic field can be reexpressed by a partial integration, using (10), as

$$-\frac{1}{4}\int dx F_{\mu\nu}F^{\mu\nu} = +\frac{1}{2}\int dx j_\mu(x)A^\mu(x) \tag{20}$$

Putting all together we have the action underlying our nonlinear equation (16), namely

$$W = \int dx \Big[\bar{\psi}(x) \left(\gamma^\mu \left(i\partial_\mu - eA_\mu^{\text{ext}} \right) - m \right) \psi(x) \tag{21}$$
$$- \frac{e^2}{2}\int dy \bar{\psi}(x)\gamma^\mu(x)\psi(x)D(x-y)\bar{\psi}(y)\gamma_\mu\psi(y) \Big]$$

We shall consider now the single electron problem in an external field.

We expand the classical field ψ into a Fourier series

$$\psi(x) = \sum\!\!\!\!\!\!\int_n \psi_n(\vec{x})e^{-iE_n t} \tag{22}$$

and shall try to determine the expansion coefficients $\psi_n(\vec{x})$ and the spectrum E_n– discrete and continous. This expansion is quite different than the one used in standard QED and quantumoptics, namely the Coulomb series expansion, for example, in the Coulomb field,

$$\psi(x) = \sum\!\!\!\!\!\!\int_n c_n(t)\psi_n^c(\vec{x})$$

Here one derives equations for the time-dependent coefficients $c_n(t)$. The idea behind is that the system has definite levels and the perturbation will cause transitions between these levels. In our formulation, due to selfenergy, there are no definite (discrete) levels as exact eigenstates of the system to begin with, but the equations will determine the spectrum. In fact it will turn out that only the ground state of the system will be a stable eigenstate followed by a continuum with spectral concentrations around the unperturbed spectrum.

If we insert the Fourier expansion into the action we obtain

$$W = \sum\!\!\!\!\!\!\int_{nm} dx \Big\{ \bar{\psi}_n(\vec{x}) e^{iE_n x^0} \left[\gamma^\mu \left(i\partial_\mu - eA_\mu^{\text{ext}} \right) - m \right] \psi_m(\vec{x}) e^{-iE_m x^0} -$$

$$- \frac{e^2}{2} \int dy \bar{\psi}_n(\vec{x}) \gamma^\mu \psi_m(\vec{x}) e^{i(E_n - E_m)x^0} D(x-y) \bar{\psi}_r(\vec{y}) \gamma_\mu \psi_s(\vec{y}) e^{i(E_r - E_s)y^0} \Big\} \tag{23}$$

Time integrations can be performed using

$$D(x-y) = -\frac{1}{(2\pi)^4} \int dk \frac{e^{-ik(x-y)}}{k^2} \tag{24}$$

and we can write the interaction part of the action entirely in terms of the Fourier components of the current

$$W_{\text{int}} = + \frac{e^2}{2} \sum\!\!\!\!\!\!\int_{nmrs} \delta(E_n - E_m + E_r - E_s) \int \bar{\psi}_n(\vec{x}) \gamma^\mu \psi_m(\vec{x})$$

$$\times \frac{e^{i\vec{k}\cdot(\vec{x}-\vec{y})}}{(E_n - E_m)^2 - \vec{k}^2} \bar{\psi}_r(\vec{y}) \gamma_\mu \psi_s(\vec{y}) d\vec{x} d\vec{y} d\vec{k} \tag{25}$$

For the exact solutions of our equations the action W will vanish identically. We will now solve the system iteratively.

To lowest order of iteration we take the field to be given by the solutions of the external field problem without the selfenergy terms, and the energies to be shifted by a small amount:

$$\psi_n(x) = \psi_n^{\text{ext}}(x)$$
$$E_n = E_n^{\text{ext}} + \Delta E_n \tag{26}$$

The first term in (22) therefore gives simply, using the orthonormality of ψ_n's,

$$W_0 = \int d\vec{x} \sum\!\!\!\!\!\!\int_{nm} \bar{\psi}_n \left(\gamma^0 E_n^{\text{ext}} - \vec{\gamma}\cdot\vec{p} - m - eA_\mu^{\text{ext}} \right) \psi_m \delta(E_n - E_m)$$

$$\Rightarrow \sum_{nm} \Delta E_n \delta(E_n - E_m) \delta nm$$

In the second term we separate the terms according to $E_n = E_m$, $E_r = E_s$ and according to $E_n = E_s$, $E_m = E_r$, the two ways of satisfying the overall δ-function. And since $W = 0$ to this order of iteration we can solve for ΔE_n. The action and the total energy of the system are related by a δ-function. Cancelling this δ-function and also the sum over n to obtain the energy shift of a fixed level n, we obtain

$$\Delta E_n = \frac{e^2}{2} \int d\vec{x} \bar{\psi}_n(\vec{x}) \gamma_\mu \psi_n(\vec{x}) P \int \frac{d\vec{k}}{(2\pi)^3} \int d\vec{y} \frac{e^{i\vec{k}\cdot(\vec{x}-\vec{y})}}{k^2} \cdot \sum\!\!\!\!\!\!\int_s \bar{\psi}_s(\vec{y}) \gamma^\mu \psi_s(\vec{y})$$

$$- \frac{e^2}{2} \sum\!\!\!\!\!\!\int_s \int d\vec{x} d\vec{y} \bar{\psi}_n(\vec{x}) \gamma_\mu \psi_s(\vec{x}) \int \frac{d\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \bar{\psi}_s(\vec{y}) \gamma^\mu \psi_n(\vec{y}) \cdot$$

$$\cdot \left[\frac{1}{E_s - E_n - k} - \frac{1}{E_s - E_n + k} \right] \tag{27}$$

$$- \frac{e^2}{2} \sum_{(s<n)} \sum\!\!\!\!\!\!\int_s \int d\vec{x} d\vec{y} \bar{\psi}_n(\vec{x}) \gamma_\mu \psi_s(\vec{x}) \int \frac{d\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \bar{\psi}_s(\vec{y}) \gamma^\mu \psi_n(\vec{y}) \cdot$$

$$\cdot \frac{i\pi}{2k} \delta(E_s - E_n - k)$$

This can be written in the form

$$\Delta E_n = -\frac{e^2}{2}\sum\!\!\!\!\!\!\int_m \frac{d\vec{k}}{(2\pi)^3}\frac{j^\mu_{nn}(\vec{k})j^{mm}_\mu(-\vec{k})}{\vec{k}^2} - \frac{e^2}{2}\sum\!\!\!\!\!\!\int_{m<n}\frac{d\vec{k}}{(2\pi)^3}j^\mu_{nm}(\vec{k})j^{mn}_\mu(-\vec{k})\frac{i\pi}{2}\delta(E_m - E_n - k)$$

$$-\frac{e^2}{2}\sum\!\!\!\!\!\!\int_m \frac{d\vec{k}}{(2\pi)^3}j^\mu_{nm}(\vec{k})j^{mn}_\mu(-\vec{k})\frac{1}{2k}\left[\frac{1}{E_m - E_n - k} - \frac{1}{E_m - E_n + k}\right] \tag{28}$$

Thus the energy shifts are entirely expressed in terms of the integrals over the Fourier spectra of currents of all states. The first term corresponds to vacuum polarization, the second to spontaneous emission, and the third term to the Lamb shift proper. In arriving at these results we have used the causal Green's function and separated the integrals into a principal and a imaginary part according to the formula

$$\frac{1}{x} = P\frac{1}{x} \pm i\pi\delta(x) \tag{29}$$

All the main QED effects are obtained here from a single expression. In fact one can also read off the anomalous magnetic moment $(g-2)$ from this expression as we shall show in Section VI.

The evaluation of these expressions is a rather laborious technical problem. We have to use relativistic Coulomb wave functions for both the discrete and continous spectrum and integrate the products of such functions and sum over the whole spectrum. We shall indicate some of these calculations and give results in Section VIII. The most important feature of the present formulation is that there are no infrared nor ultraviolet divergences.

The spontaneous emission term in Eq. (27) has been exactly evaluated[12]. We have now complete relativistic spontaneous decay rates for all hydrogenic states[13]. Table II shows some of these results.

TABLE II. Decay rates (s^{-1}) in hydrogen and muonium

Transition	Hydrogen	Muonium
$2S_{1/2} \to 1S_{1/2}$	2.4964×10^{-6}	2.3997×10^{-6}
$2S_{1/2} \to 1P_{1/2}$	5.194×10^{-10}	5.172×10^{-10}
$2P_{1/2} \to 1S_{1/2}$	2.0883×10^{8}	2.0794×10^{8}
$2P_{3/2} \to 1S_{1/2}$	4.1766×10^{8}	4.1587×10^{8}
$2P \to 1S_{1/2}$	6.2649×10^{8}	6.2382×10^{8}

The vacuum polarization term has also been evaluated analytically[14] to lowest order term in $\alpha(Z\alpha)^4$. This is the most divergent term in perturbative QED and vanishes in the nonrelativistic limit.

The Lamb shift term which correctly reduces to the standard expressions in the dipole approximation has also been shown to be finite and will be evaluated in closed form[15].

In all these calcuations, since we are using Coulomb wave fuctions instead of the plane waves, the individual integrals are all finite. The summation over all the discrete and continous levels are done by means of the relativistic Coulomb Green's functions.

V. QUANTUMELECTRODYNAMICS OF THE RELATIVISTIC TWO-BODY SYSTEM

One of the most important and perhaps unexpected features of the selffield formulation of quantumelectrodynamics turned out to be a nonperturbative treatment of two and many body systems in closed from. It is well known that bound state problems cannot be treated in perturbative QED starting from first principles. Instead one begins from a Schrödinger or Dirac-like equation obtained from some approximation to the Bethe-Salpeter relations and then calculates the perturbation diagrams to the bound state solutions of these equations. What one really needs is a genuine two-body relativistic equation which includes all the radiative terms as well as all the recoil corrections at once. We shall now discuss the principles of this theory.

In nonrelativistic quantum theory the many body problem is formulated in configuration space by a wave equation with pair potentials $v_{ij}(x_i - x_j)$ of the form

$$\left(\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \ldots V_{12} + V_{13} + V_{23} + \ldots\right)\psi(x_1, \ldots, x_n; t) = i\hbar\frac{\partial\psi}{\partial t}$$

This a priori not obvious. We may also think that each particle has its own field $\psi(x)$ and satisfy a wave equation with a potential coming from the charge distribution of the other particles. For two particles, for example, we would have the coupled Hartree-type equations

$$i\hbar\frac{\partial\psi_1(\vec{x}_1,t)}{\partial t} = \left(-\frac{\hbar^2}{2m_1}\Delta + \int\frac{\psi_2^*(x_2,t)\psi_2(x_2,t)}{|\vec{x}_2 - \vec{x}_1|}d\vec{x}_2\right)\psi_1(\vec{x}_1,t)$$

$$i\hbar\frac{\partial\psi_2(\vec{x}_2,t)}{\partial t} = \left(-\frac{\hbar^2}{2m_2}\Delta + \int\frac{\psi_1^*(\vec{x}_1,t)\psi_1(\vec{x}_1,t)}{|\vec{x}_1 - \vec{x}_2|}d\vec{x}_1\right)\psi_2(\vec{x}_2,t)$$

These two formulations are closely related but not identical. We shall see that they correspond to two different types of variational principles and actually describe two different types of physical situations. Quantum theory has a separate new postulate for two or more particles, namely that the state space is the tensor product of one particle state spaces. This leads immediately to the first formulation in configuration space. Such combined systems are called in the axiomatic of quantum theory "nonseparated" systems with all the nonlocal properties of quantum theory. But this postulate does not apply universally. There are other systems, namely the "separated" systems, which are described by the second type of equations. For example, for the system hydrogen molecule the two protons are separated, whereas the two electrons are nonseparated. The superposition principle holds for the nonseparated systems only. We shall now see how all this comes about from two different basic variational princples in the relativistic case (the nonrelativistic case is similar).

Consider a number of matter fields $\psi_1(x), \psi_2(x)\ldots$ The action of these fields interacting via the electromagnetic field is

$$W = \int dx\left\{\sum_k \bar{\psi}_k\left(\gamma^\mu i\partial_\mu - m_k\right)\psi_k - j_\mu(x)A^\mu(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\} \tag{30}$$

where the current j^μ is the sum of Dirac currents for each field

$$j^\mu(x) = \sum_k e_k\bar{\psi}_k(x)\gamma^\mu\psi_\mu(x) \tag{31}$$

Again in the gauge $A^{\mu},_{\mu} = 0$ we obtain the equations for the electromagnetic field as

$$\Box A_{\mu} = j_{\mu} = \sum_{k} j_{\mu}^{(k)} \tag{32}$$

with the solution

$$A_{\mu}(x) = \int dy D(x-y) j_{\mu}(x). \tag{33}$$

If we insert this into the action both in the $j^{\mu} \cdot A_{\mu}$ term as well as in the term $-(1/4)F_{\mu\nu}F^{\mu\nu}$, and using the identity (20), we obtain

$$W = \int dx \sum_{k} \bar{\psi}_k \left(\gamma^{\mu} i\partial_{\mu} - m_k\right)\psi_k - \sum_{k,\ell} \frac{1}{2} \int dx dy j_{\mu}^{(k)}(x) D(x-y) j_{(\ell)}^{\mu}(y) \tag{34}$$

The interaction action is a sum of current-current interactions containing both the mutual interaction terms, e.g.

$$-\frac{e_1 e_2}{2} \int dx dy \bar{\psi}_1(x)\gamma^{\mu}\psi_1(x) D(x-y) \bar{\psi}_2(y)\gamma_{\mu}\psi_2(y) - (1 \leftrightarrow 2)$$

and the self interaction terms like

$$-\frac{e_1^2}{2} \int dx dy \bar{\psi}_1(x)\gamma^{\mu}\psi_1(x) D(x-y) \bar{\psi}_1(y)\gamma_{\mu}\psi_1(y)$$

If we vary this action with respect to each field ψ_k separately we obtain coupled nonlinear equations. For example for two particles

$$\begin{aligned}
\left(\gamma^{\mu} i\partial_{\mu} - m_1\right)\psi_1 &= \frac{e_1 e_2}{2}\gamma^{\mu}\psi_1 \int dy D(x-y)\bar{\psi}_2(y)\gamma_{\mu}\psi_2(y) \\
&\quad + \frac{e_1^2}{2}\gamma^{\mu}\psi_1 \int dy D(x-y)\bar{\psi}_1(y)\gamma_{\mu}\psi_1(y) \\
\left(\gamma^{\mu} i\partial_{\mu} - m_2\right)\psi_2 &= \frac{e_1 e_2}{2}\gamma^{\mu}\psi_2 \int dy D(x-y)\bar{\psi}_1(y)\gamma_{\mu}\psi_1(y) \\
&\quad + \frac{e_2^2}{2}\gamma^{\mu}\psi_2 \int dy D(x-y)\bar{\psi}_2(y)\gamma_{\mu}\psi_2(y)
\end{aligned} \tag{35}$$

Next let us define a composite field Φ by

$$\Phi(x_1, x_2) = \psi_1(x_1)\psi_2(x_2) \tag{36}$$

This is a 16-component spinor field. We can rewrite our action (34) entirely in terms of the composite field. This is straightforward in the mutual interaction terms. In the kinetic energy and selfinteraction terms we multiply suitable by normalization factors. For example for the first kinetic energy term we get

$$\int dx_1 \bar{\psi}_1(x_1)\left(\gamma^{\mu} i\partial_{\mu} - m_1\right)\psi_1(x_1) \cdot \int d\sigma_2 \bar{\psi}_2(x_2)\gamma \cdot n\psi_2(x_2)$$

where $d\sigma_2 n^\mu = d\sigma_2^\mu$ is a 3-dimensional volume element perpendicular to the normal n^μ. Similarly for the other kinetic energy term. The selfenergy terms need two such normalization factors. The resultant action in terms of the composite field is then

$$W = \Big[\int dx_1 d\sigma_2 \bar{\Phi}(x_1 x_2)(\gamma^\mu \pi_{1\mu} - m_1) \otimes \gamma \cdot n \Phi(x_1 x_2)$$
$$+ \int dx_2 d\sigma_1 \bar{\Phi}(x_2 x_1) \gamma \cdot n \otimes (\gamma^\mu \pi_{2\mu} - m_2) \Phi(x_2 x_1) \tag{37}$$
$$- e_1 e_2 \int dx_1 dx_2 \bar{\Phi}(x_1 x_2) \gamma^\mu \otimes \gamma_\mu D(x_1 - x_2) \Phi(x_1 x_2) \Big]$$

The generalized canonical momenta $\pi_{i\mu}$ are given further below. Here and through the rest of the paper we shall write spin matrices in the form of tensor products $\otimes$, the first factor always referring to the spin space of particle 1, the second to particle 2. We shall give the selfenergy terms explicitely below.

Now our second variational principle is that the action be stationary not with respect to the variations of the individual fields but with respect to the total composite field only. This is a weaker condition than before and leads to an equation for Φ in configuration space. For bound state problems only the symmetric Green's function contributes and it contains a $\delta(x^2)$-function which we decompose relative to the space-like surface with normal n^μ as follows

$$\delta(r^2) = \frac{\delta[(r \cdot n) - r_\perp) \pm \delta[(r \cdot n) + r_\perp)}{2 r_\perp}, \; r_\perp = [(r \cdot n)^2 - r^2]^{1/2} \tag{38}$$

where $r_\perp$ is a relativistic three dimensional distance which for $n = (1000)$ reduces to the ordinary distance r. All the integrals in the action (37) are 7-dimensional. For covariance purposes it is necessary to have the vector n^μ. It tells us how to choose the time axis. The vector n is also present, in principle, in the one-body Dirac equation but we usally do not write it when discussing the solutions, but automatically choose it to be $n = (1000)$, i.e., the rest frame. The final form of our two-body equation is then

$$\left\{ (\gamma^\mu \pi_1^\mu - m_1) \otimes \gamma \cdot n + \gamma \cdot n \otimes (\gamma^\mu \pi_2^\mu - m_2) - e_1 e_2 \frac{\gamma^\mu \otimes \gamma_\mu}{r_\perp} \right\} \Phi(x_1 x_2) = 0 \tag{39}$$

where now the selfpotentials are inside the generalized momenta

$$\pi_i^\mu = p_i^\mu - e_i A_i^{\mu \text{self}} - e_i A_i^{\mu \text{ext}} \tag{40}$$

with

$$A_{\mu,1}^{\text{self}}(x) = \frac{e_1}{2} \int dz d\sigma_u D(x - z) \bar{\Phi}(z, u) \gamma_\mu \otimes \gamma \cdot n \Phi(z, u)$$
$$A_{\mu,2}^{\text{self}}(x) = \frac{e_2}{2} \int d\sigma_z du D(x - u) \bar{\Phi}(z, u) \gamma \cdot n \otimes \gamma_\mu \Phi(z, u) \tag{41}$$

We note that the last term in (39) can also be put into the potential A_μ, one half for each particle; $\frac{1}{2} e_2 \frac{1 \otimes \gamma_\mu}{r}$ and $\frac{1}{2} e_1 \frac{\gamma_\mu \otimes 1}{r}$, respectively.

The self potentials are nonlinear integral expressions. The arguments of Φ consist of seven variables because $\Phi(x_1, x_2)$ is different from zero only if $(x_1 - x_2)$ is lightlike; only then there is a communication between the particles. This means that

we have one time-variable and three space variables for each particle. We see this more cleary if we introduce center of mass and relative variables according to

$$\begin{aligned} \Pi &= \pi_1 + \pi_2 \ , \quad \pi = \pi_1 - \pi_2 \\ x &= x_1 - x_2 \ , \quad X = x_1 + x_2 \end{aligned} \tag{42}$$

Then equation (39), without the selffield terms for simplicity, becomes

$$\left\{ \Gamma^\mu \Pi_\mu + k^\mu \pi_\mu - \frac{e_1 e_2}{r_\perp} \gamma^\mu \otimes \gamma_\mu - m_1 I \otimes \gamma \cdot n - m_2 \gamma \cdot n \otimes I \right\} \Phi = 0 \tag{43}$$

where we have introduced

$$\Gamma^\mu = \frac{1}{2}(\gamma^\mu \otimes \gamma \cdot n + \gamma \cdot n \otimes \gamma^\mu)$$

and

$$k^\mu = \frac{1}{2}(\gamma^\mu \otimes \gamma \cdot n - \gamma \cdot n \otimes \gamma^\mu) \tag{44}$$

We see now that $k \cdot n = 0$, i.e., the component of k^μ parallel to n^μ vanishes which means that the component of the relative momentum π_μ parallel to n_μ drops out of the equation automatically. For $n = (1000)$, in particular, we have

$$\left\{ \Gamma^0 \Pi_0 - \vec{\gamma} \cdot \vec{\Pi} - \vec{k} \cdot \vec{\pi} - \frac{e_1 e_2}{r} \gamma^\mu \otimes \gamma_\mu - m_1 I \otimes \gamma_0 - m_2 \gamma_0 \otimes I \right\} \Phi = 0 \tag{45}$$

Thus we have only one time variable conjugate to the center of mass energy Π_0 and three degrees of freedom for the center of mass momentum $\vec{\Pi}$ and three degrees of freedom for the relative momentum $\vec{\pi}^0$; π_0 does not enter, as it should be so on physical grounds. In contrast the Bethe-Salpeter equation has two time coordinates. Since Π_0 is the "Hamiltonian" of the system we obtain, by multiplying (45) by Γ_0^{-1} the Hamiltonian form of the two-body equation

$$\Pi_0 \Phi = \left\{ \vec{\alpha} \cdot \vec{\Pi} + (\vec{\alpha}_1 - \vec{\alpha}_2) \cdot \vec{\pi} + \frac{e_1 e_2}{r}(1 - \vec{\alpha}_1 \cdot \vec{\alpha}_2) + m_1 \beta_1 \cdot I + m_2 I \cdot \beta_2 \right\} \Phi$$

where we have defined

$$\vec{\alpha} \equiv \frac{1}{2}(\vec{\alpha}_1 + \vec{\alpha}_2); \ \vec{\alpha}_i = \gamma_i^0 \vec{\gamma}_i, \beta_i = \gamma_{0i}, \ i = 1,2 \tag{46}$$

Our two-body equation has the form of a generalized Dirac equation, now a 16-component wave equation. In fact it reduces to the one-body Dirac equation in the limit when one of the particles is heavy.

The above developments are completely relativistic and covariant. The physical results are independent of the vector n although a vector n must appear for manifest covariance. Thus recoil corrections are included to all orders. Further interesting properties of the equation, beside being a one-time relativistic equation, are that relative and center of mass terms in the Hamiltonian are additive, and radial and angular parts of the relative equation are exactly separable. It has also a non-relativistic limit to the two-body Schrödinger equation. We shall discuss numerical results in Section VIII.

VI. THE INTERPRETATION OF NEGATIVE ENERGY STATES

It is often stated that only in second quantized field theory can one have an adequate description of antiparticles and negative energy solutions where one changes the roles of the creation and annihilation operators for the negative energy solutions. We shall now show that there is also a consistent way of dealing with the negative energy solutions and antiparticles in the Dirac equation as a classical field theory and elaborate how we obtain the annihilation potential in positronium, for example.

There are actually not one but two Diract equations

$$\begin{aligned}(\gamma \cdot p - m)\psi_I &= 0 \\ (\gamma \cdot p + m)\psi_{II} &= 0\end{aligned} \tag{47}$$

obtained from the factorization of the Klein-Gordon operator, for example. By convention we just peak one and work with the complete set of solutions of this equation. Now the negative energy solutions of ψ_I coincide with the positive energy solutions of ψ_{II}. Furthermore in the presence of the electromagnetic field with minimal coupling we have the two equations

$$\begin{aligned}(\gamma \cdot (p - eA) - m)\,\psi_I &= 0 \\ (\gamma \cdot (p - eA) + m)\,\psi_{II} &= 0\end{aligned} \tag{48}$$

and we can easily prove that

$$\psi_I(-p, -e) = \psi_{II}(p, e) \tag{49}$$

that is the negative energy momentum solutions of ψ_I coincide with the positive energy solutions of ψ_{II} of opposite charge. Therefore we should consider positive energy solutions of both equations as physical particles. The total number of such physical solutions is the same as the total number of both positive and negative energy solutions of a single Dirac equation.

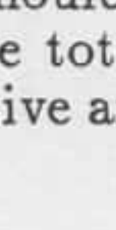
With this interpretation we obtain quite naturally the annihilation diagrams and annihilation potentials between particles and antiparticles. Consider our interaction action

$$\int dx dy \bar{\psi}_1(x)\gamma^\mu \psi_1(x) D(x - y)\bar{\psi}_2(y)\gamma_\mu \psi_2(y)$$

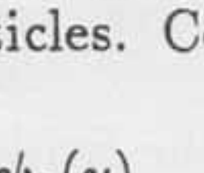
Here the classical fields $\psi_i(x)$ contain all positive and negative energy solutions according to our general expansion (22). Separating positive and negative energy solutions as

$$\psi_n(x) = \psi_{E_N > o} + \psi_{E_N < o} \equiv \psi_n^+ + \psi^-$$

and inserting it into the action we get 16 terms. In the limiting case of the lowest order scattering, where we replace the fields by plane wave solutions, we have essentially two distinct types of vertices at each point x or y, namely

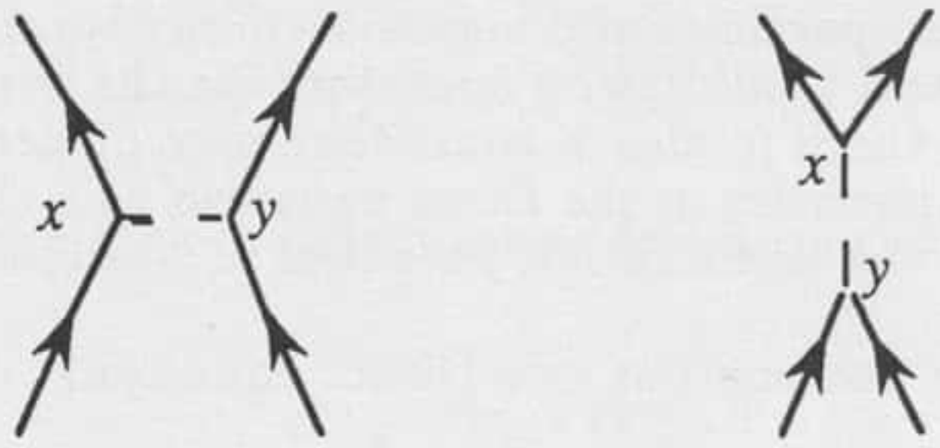
$$\bar{\psi}^+(x)\gamma_\mu \psi^+(x) \qquad \text{  } \tag{50}$$

and

$$\bar{\psi}^+(x)\gamma_\mu \psi^-(x) \qquad \text{  }$$

In the second case we have used our interpretation of the negative energy solutions as the antiparticles with reversed energy-momentum p_μ. The complete interaction action to this order consists of all combinations of these two vertices located at x and

y for particles 1 and 2 multiplied with the Green's function $D(x-y)$. Of these 16 terms some cannot be realized because of the overall energy-momentum conserving δ-function, $\delta(p_1+p_2-p_3-p_4)$, and we are left with two disctinct types of terms



(51)

plus the same terms with particles and antiparticles interchanged. This result agrees with the standard QED. But we shall go a bit further and apply it to bound state problems in Section VII after a discussion of the case of identical particles.

Identical Particles

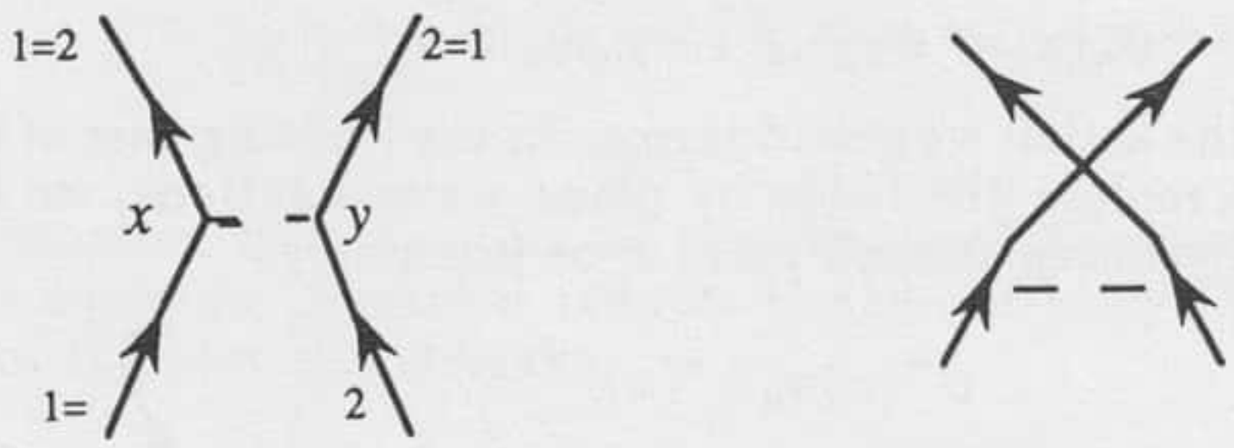
For two identical particles we use the postulate of the first quantized quantum theory that the field is symmetric or antisymmetric under the interchange of all dynamical variables of identical particles. In our formulation we go back to the original action principle and assume that the current j_μ is antisymmetric in the two fields

$$j_\mu = \frac{1}{2}e\left(\bar{\psi}_1\gamma_\mu\psi_2 - \bar{\psi}_2\gamma_\mu\psi_1\right), \quad e_1 = e_2 = e \tag{52}$$

This implies in the interaction action

$$W_{\text{int}} = \frac{1}{4}e^2\left[\int dxdy\bar{\psi}_1(x)\gamma_\mu\psi_2(x)D(x-y)\bar{\psi}_1(y)\gamma^\mu\psi_2(y) - \int dxdy\bar{\psi}_1\gamma_\mu\psi_2 D(x-y)\bar{\psi}_2\gamma^\mu\psi_1 + (1\leftrightarrow 2)\right] \tag{53}$$

and again when the fields are expanded we see that identical particles with exactly the same wave functions i.e., the same quantum numbers or the same state, will not interact and that in the lowest approximation we will get besides the direct interaction also an exchange term as shown in the following diagrams



Finally we combine the two effects, identical particles and particle-antiparticle properties, to discuss systems like electron-positron complex and positronium. According to our discussion this system is just a part of the larger electron-electron system taking into account the interpretation of the negative energy levels and the identicity of the particles.

VI. CALCULATION OF THE ANOMALOUS MAGNETIC MOMENT $(g-2)$

We show now that our basic interaction action also contains besides Lamb shift, spontaneous emission and vacuum polarization also the anomalous magnetic moment in the same single expression. We shall also introduce at this occasion the more general four-dimensional energy-momentum Fourier expansion instead of the energy Fourier expansion (22) which was appropriate for the fixed external field problem.

The interaction action is given by

$$W_{\text{int}} = -\frac{e^2}{2}\int dxdy j^{\mu}(x)D(x-y)j_{\mu}(y) \tag{54}$$

We expand the fields as four dimensional Fourier integrals

$$\psi(x) = \int dp e^{-ipx}\psi(p) \tag{55}$$

and insert it into the action

$$W_{\text{int}} = -\frac{e^2}{2}\frac{1}{(2\pi)^4}\int dxdydkdpdqdrds\bar{\psi}(p)\gamma^{\mu}\psi(p)\frac{e^{-ik(x-y)}}{k^2+i\varepsilon}\bar{\psi}(r)\gamma_{\mu}\psi(s)$$
$$\times e^{i(p-q)x+(r-s)y}$$

which can be written as

$$W_{\text{int}} = -\frac{e^2}{2}(2\pi)^4\int dpdqdrds j^{\mu}(p,q)\frac{1}{(r-s)^2+i\varepsilon}j_{\mu}(r,s)\delta(p-q+r-s) \tag{56}$$

where $j^{\mu}(p,q)$ stand for the double Fourier transform

$$j^{\mu}(p,q) = (2\pi)^8\int dxdy\bar{\psi}(x)\gamma^{\mu}\psi(y)e^{-ipx}e^{iqy} \tag{57}$$

The δ-function arises from the x,y and k-integrations. We separate the action into two terms to satisfy the δ-function

$$\begin{aligned}(i)\quad & p=q, \text{ hence } r=s\\ (ii)\quad & p=s, \text{ hence } q=r\end{aligned} \tag{58}$$

Again as before the first corresponds to vacuum polarization; the second term contains Lamb shift and spontaneous emission as real and imaginary parts of the energy shift ΔE. It also contains the anomalous magnetic moment as the coefficient of the magnetic part of the Lamb shift for any external field. For the calculation of $(g-2)$ it is thus not necessary to solve a problem with an external magnetic field or to solve any external problem for that matter.

The second term with (ii) can be written as

$$W_{\text{int}}^{(ii)} = -\frac{e^2}{2}\int dxdydpdqD(x-y)j^{\mu}(q,p)j_{\mu}(q,p)e^{ipx}e^{i(y-x)}q \tag{59}$$

Here we recognize the c-number electron propagator-function $S(x-y)$

$$\int dq\psi(q)\bar{\psi}(q)e^{i(y-x)q} \equiv S(x-y) \tag{60}$$

which satisfy the inhomogeneous wave equation

$$[\gamma^\mu(p_\mu - eA_\mu) - m] = i\delta(x-y) \tag{61}$$

Let us also take the Fourier transform of S

$$S(x-y) = \frac{1}{(2\pi)^4}\int dp e^{-ip(x-y)}S(p) \tag{62}$$

Then the action (59) becomes

$$W_{\text{int}}^{(ii)} = -\frac{e^2}{2}i\int dpdP\bar{\psi}(p)\frac{\gamma^\mu S(p)\gamma_\mu}{(p-P)^2+i\varepsilon}\psi(p) \tag{63}$$

It is related to the energy, more precisely to a mass shift by an overall δ-function and we can write

$$\Delta E = \frac{W_{\text{int}}^{(ii)}}{(2\pi)^4} = \int dp\bar{\psi}(p)\Delta M(p)\psi(p) \tag{64}$$

where we have introduced an effective *mass matrix* by

$$\Delta M(p) = \frac{e^2}{2}\frac{i}{(2\pi)^4}\int ds\frac{\gamma^\mu S(p-s)\gamma_\mu}{s^2-i\varepsilon} \tag{65}$$

It remains now to evaluate the mass matrix ΔM. First we expand the Green's function or propagator in an external field as follows

$$\frac{1}{\gamma^\mu(p_\mu - eA_\mu) - m} = \frac{\not{p} - e + m}{p^2-m^2} + 2e\frac{p\cdot A(\not{p}+m)}{(p^2-m^2)^2} - ie\frac{(\not{p}+m)\gamma^\mu\gamma^\nu F_{\mu\nu}}{(p^2-m^2)^2} + \dots$$

where

$$\not{p} \equiv \gamma^\mu p_\mu,\ \not{A} = \gamma^\mu A_\mu,\ p\cdot A = p^\mu A_\mu. \tag{66}$$

It turns out that only the third term which is gauge invariant gives a nonvanishing contribution to lowest order in α: the terms containing A_μ give vanishing contributions. The mass operator becomes

$$\Delta M(p) = \frac{e^2}{2}\frac{i}{(2\pi)^4}(-ie)\int ds\frac{\gamma^\mu\left((p-s)-m\right)\gamma^\lambda\gamma^\varphi F_{\lambda\varphi}\gamma_\mu}{s^2\left((p-s)^2-m^2\right)^2} \tag{67}$$

The integrals can be performed giving[16]

$$\Delta M(p) = \frac{e^2}{2}\frac{i}{(2\pi)^4}(-ie)\frac{2}{m}\int_0^1 dy(1-y)\sigma^{\mu\nu}F_{\mu\nu}$$

and finally

$$\Delta M(p) = -\frac{\alpha}{2\pi}\frac{e}{2m}\sigma_{\mu\nu}F_{\mu\nu} = -\frac{\alpha}{2\pi}\frac{e}{2m}(\vec{\sigma}\cdot\vec{B} + i\vec{\alpha}\cdot\vec{E}) \tag{68}$$

Thus we recognize the anomalous magnetic moment to this order in front of the $\vec{\sigma} \cdot \vec{B}$ term for any field as we have mentioned.

If we insert the mass operator into the energy shift formula (64) we can evaluate it as the expectation value of the operator $\vec{\sigma} \cdot \vec{B} + i\vec{\alpha} \cdot \vec{E}$. For relativistic Coulomb problem for example the magnetic part of the Lamb shift can be analytically evaluated exactly.[17]

This way of calculating the anomalous magnetic moment also shows now how to calculate higher order terms. We must take more terms in the Green's function expansion (66). This may be much simpler than the diagrammatic method of perturbative QED where there are already 891 Feynman diagrams in the order $(\alpha/\pi)^4$.

VII. COVARIANT ANALYSIS OF RADIATIVE PROCESSES FOR TWO-BODY SYSTEMS

In this section we discuss how to treat radiative processes, like Lamb shift, etc., for a system like positronium or muonium beyond the naive reduced mass method. As mentioned above the action formalism is more convenient than the equations of motion.

We go back to our covariant 2-body action (37) and separate center of mass and relative coordinates and momenta according to

$$\begin{aligned}
r &= x_1 - x_2 & x_1 &= R + \frac{1}{2}r & P &= p_1 + p_2 \\
R &= \frac{1}{2}(x_1 + x_2) & x_2 &= R - \frac{1}{2}r & p &= \frac{1}{2}(p_1 - p_2) \\
q &= z - u & z &= Q + \frac{1}{2}q & p_1 &= \frac{1}{2}P + p \\
Q &= \frac{1}{2}(z + u) & u &= Q - \frac{1}{2}q & p_2 &= \frac{1}{2}P - p
\end{aligned} \tag{69}$$

All quantities here are four-vectors. Then the action becomes

$$\begin{aligned}
W = \int dRdq\bar{\Phi}(R,r)\Bigg\{ & \left[\gamma \cdot \left(\frac{1}{2}P + p\right) - m_1\right] \otimes \gamma \cdot n + \gamma \cdot n \otimes \left[\gamma \cdot \left(\frac{1}{2}P - p\right) - m_2\right] \\
& - e_1 e_2 D(r) - \frac{e_1^2}{2} \int dQdq\gamma_\mu \otimes \gamma \cdot n D\left(R - Q + \frac{1}{2}(r-q)\right) \bar{\Phi}(Q,q)\gamma^\mu \otimes \gamma \cdot n\Phi(Q,q) \\
& - \frac{e_2^2}{2} \int dQdq\gamma \cdot n \otimes \gamma_\mu D\left(R - Q - \frac{1}{2}(r-q)\right) \bar{\Phi}(Q,q)\gamma \cdot n \otimes \gamma^\mu \Phi(Q,q)\Bigg\} \Phi(R,r)
\end{aligned} \tag{70}$$

In the absence of a fixed external field the system is translationally invariant and the generalization of the Fourier expansion (22) is the four dimensional Fourier transform of the composite field $\Phi(R,r)$ which has actually one time variable, $\Phi(R,r_\perp)$, the relative coordinates is a 3-vector $r_\perp$ perpendicular to n.

$$\Phi(R,r_\perp) = \int \frac{d^4P}{(2\pi)^4} e^{iPR} \psi(P,r_\perp)$$

We insert this expansion everywhere in our action and obtain

$$W = \int dRdr_\perp \int \frac{dP_n}{(2\pi)^4}\frac{dP_m}{(2\pi)^4}\bar{\psi}(P_n, r_\perp)e^{-iP_nR}\Bigg\{ \left[\Gamma_\mu P^\mu + \mathcal{L}_{rel}(r_\perp, p)\right] e^{iP_mR}$$

$$-\frac{1}{2}\int \frac{dP_r}{(2\pi)^4}\frac{dP_s}{(2\pi)^4}dQdq_\perp dk\Bigg[\frac{e_1^2}{2}\frac{e^{-ik[R-Q+\frac{1}{2}(r_\perp-q_\perp)]}}{k^2}\gamma^\mu \otimes \gamma\cdot n\bar{\psi}(P_r, q_\perp)e^{-iP_rQ}\gamma_\mu$$

$$\otimes \gamma\cdot n e^{iP_sQ}\psi(P_s, q_\perp)$$

$$+\frac{e_2^2}{2}\frac{e^{-ik[R-Q-\frac{1}{2}(r_\perp-q_\perp)]}}{k^2}\gamma\cdot n\otimes\gamma^\mu\bar{\psi}(P_r, q_\perp)e^{-iP_rQ}\gamma\cdot n\otimes\gamma_\mu e^{iP_sQ}$$

$$\times\psi(P_s, q_\perp)\Bigg]\Bigg\}\psi(P_m, r_\perp) \tag{71}$$

where $\mathcal{L}_{rel}$ is the Lagrangian of the relative motion and is given by

$$\mathcal{L}_{rel}(r_\perp, p) = (\gamma^\mu p_\mu - m_1)\otimes\gamma\cdot n + \gamma\cdot n\otimes(-\gamma^\mu p_\mu - m_2) - \frac{e_1e_2}{r_\perp}\gamma^\mu\otimes\gamma_\mu \tag{72}$$

and

$$\Gamma_\mu = \frac{1}{2}(\gamma_\mu\otimes\gamma\cdot n + \gamma\cdot n\otimes\gamma_\mu) \tag{73}$$

The result of performing the R and Q-integrations, letting

$$k_\mu = \gamma_\mu\otimes\gamma\cdot n - \gamma\cdot n\otimes\gamma_\mu \tag{74}$$

is

$$W = \int \frac{dP_n}{(2\pi)^4}dP_m dr_\perp\bar{\psi}(p_n, r_\perp)\Bigg\{\Bigg[\Gamma_\mu P^\mu + k^\mu p_\mu - m_1 I\otimes\gamma\cdot n - m_2\gamma\cdot n\otimes I - \frac{e_1e_2}{r_-}\gamma^\mu\otimes\gamma_\mu\Bigg]$$

$$\times\delta(P_n - P_m) - \frac{1}{2}\int\frac{dP_r}{(2\pi)^4}dP_s\frac{dk}{(2\pi)^4}dr_\perp dq_\perp\delta(P_n - P_m - k)\delta(P_r - P_s - k)\Big[e_1^2e^{-i\frac{1}{2}k(r_\perp - q_\perp)}$$

$$\times\gamma^\mu\otimes\gamma\cdot n\bar{\psi}(P_r, q_\perp)\gamma_\mu\otimes\gamma\cdot n + \epsilon_2^2\epsilon^{i\frac{1}{2}k(r_\perp-q_\perp)}\gamma\cdot n\otimes\gamma^\mu\bar{\psi}(P_r, q_\perp)\gamma\cdot n\otimes\gamma^\mu\Big]\psi(P_S, q_\perp\Bigg\}$$

$$\psi(P_m, r_\perp) \tag{75}$$

Introducing the form factors

$$\overset{(1)}{T^\mu}_{nm}(k) \equiv \int dr_\perp\bar{\psi}(P_n, r_\perp)e^{\frac{i}{2}kr_\perp}\gamma^\mu\otimes\gamma\cdot n\psi(P_m, r_\perp)$$

and (76)

$$\overset{(2)}{T^\mu}_{nm} = \int dr_\perp\bar{\psi}(P_n, r_\perp)e^{-\frac{i}{2}kr_\perp}\gamma\cdot n\otimes\gamma^\mu\psi(P_m, r_\perp)$$

for the two particles, we can write the action in the compact form

$$W = \sum\!\!\!\!\!\!\int_{nm} dr_\perp\bar{\psi}(P_n, r_\perp)\left[\Gamma_\mu P^\mu + \mathcal{L}_{rel}\right]\psi(P_m, r_\perp)\delta(P_n - P_m)$$

$$-\frac{1}{2}\sum\!\!\!\!\!\!\int_{nm}\sum\!\!\!\!\!\!\int_{rs}\frac{dk}{(2\pi)^4}\frac{1}{k^2}\left[e_1^2\overset{(1)}{T^k}_{nm}\overset{(1)}{T^{rs}}_{\mu} + e_2^2\overset{(2)}{T^\mu}_{nm}\overset{(2)}{T^{rs}}_{\mu}\right]\delta(P_n - P_m + k)\delta(P_r - P_s - k) \tag{77}$$

where

$$\mathcal{L}_{rel}(r_\perp) = k^\mu p_\mu - m_1 I \otimes \gamma \cdot n - m_2 \gamma \cdot n \otimes I - \frac{e_1 e_2}{r_\perp} \gamma^\mu \otimes \gamma_\mu$$

Note that $k^\mu n_\mu = 0$.

Now we can perform the k^0-integration, and without loss of generality set $n = (1000)$, and obtain

$$\begin{aligned} W = & \sum\!\!\!\!\!\!\int_{nm} \delta(E_n - E_m)\delta(\vec{P}_n - \vec{P}_m) d\vec{r} \bar{\psi}_n(\vec{P}_n, \vec{r})[\Gamma_0 P^0 - \vec{\Gamma} \cdot \vec{P} + \mathcal{L}_{rel}]\psi_m(\vec{P}_m, \vec{r}) \\ & - \frac{1}{2} \sum\!\!\!\!\!\!\int_{nm} \sum\!\!\!\!\!\!\int_{rs} \frac{d\vec{k}}{(2\pi)^4} \frac{1}{\omega_{nm}^2 - \vec{k}^2} \Big[e_1^2 \overset{(1)}{T^\mu}_{nm}(\omega_{nm}, \vec{k}) \overset{(1)}{T^{rs}}_{\mu}(\omega_{rs}, \vec{k}) \\ & + e_2^2 \overset{(2)}{T^\mu}_{nm}(\omega_{nm}, \vec{k}) \overset{(2)}{T^{rs}}_{\mu}(\omega_{rs}, \vec{k}) \Big] \delta(\omega_{nm} + \omega_{rs}) \delta(\vec{P}_n - \vec{P}_m + \vec{k}) \delta(\vec{P}_r - \vec{P}_s - \vec{k}) \end{aligned} \tag{78}$$

Now we look at the selfinteraction terms only and expand the denominator

$$\begin{aligned} W^{\text{self}} = & \frac{1}{2} \sum\!\!\!\!\!\!\int_{nm} \sum\!\!\!\!\!\!\int_{rs} \frac{d\vec{k}}{(2\pi)^4} \left[e_1^2 \overset{(1)}{T^\mu}_{nm}(\omega_{nm}, \vec{k}) \overset{(1)}{T^{rs}}_{\mu}(-\omega_{nm}, -\vec{k}) + e_2^2 \overset{(2)}{T^\mu}_{nm}(\omega_{nm}, \vec{k}) \overset{(2)}{T^{rs}}_{\mu}(-\omega_{nm}, -\vec{k}) \right] \\ & \times \delta(\omega_{nm} + \omega_{rs}) \left\{ (\delta(\omega_{rs} - k) + \delta(\omega_{rs} + k)) \frac{2\pi i}{2k} + P \frac{1}{ik} \left(\frac{1}{\omega_{rs} - k} - \frac{1}{\omega_{rs} + k} \right) \right\} \\ & \delta(\vec{P}_n - \vec{P}_m + \vec{k}) \delta(\vec{P}_n - \vec{P}_m + \vec{P}_r - \vec{P}_s) \end{aligned} \tag{79}$$

where P stands for the principal value of the integral and $\sum\!\!\!\!\!\!\int$ means a summation over discrete states and an integration over the continuum states. As in the case of the Coulomb problem and $(g-2)$-calculation, we separate the two terms corresponding to

$$\text{(a) } n = m, \text{ hence } r = s$$

and

$$\text{(b) } n = s, \text{ hence } m = r$$

and dictated by the δ-functions and obtain finally

$$\begin{aligned} W^{\text{self}} = & -\frac{1}{2} \sum\!\!\!\!\!\!\int_{ns} \frac{d\vec{k}}{(2\pi)^4} \left\{ e_1^2 \overset{(1)}{T^\mu}_{nn}(0, \vec{k}) \overset{(1)}{T^{ss}}_{\mu}(0, -\vec{k}) + e_2^2 \overset{(2)}{T^\mu}_{nn}(0, \vec{k}) \overset{(2)}{T^{ss}}_{\mu}(0, -\vec{k}) \right\} \\ & \times \left\{ \frac{i\pi}{k} (\delta(k) + \delta(-k)) + \frac{1}{2k} P \left(-\frac{2}{k} \right) \right\} \delta(\vec{P}_n - \vec{P}_m + \vec{k}) \delta(\vec{P}_n - \vec{P}_m + \vec{P}_r - \vec{P}_s) \\ & - \frac{1}{2} \sum\!\!\!\!\!\!\int_{nm} \frac{d\vec{k}}{(2\pi)^4} \left\{ e_1^2 \overset{(1)}{T^\mu}_{nm}(\omega_{nm}, \vec{k}) \overset{(1)}{T^{rs}}_{\mu}(-\omega_{nm}, -\vec{k}) + e_2^2 \overset{(2)}{T^\mu}_{nm}(\omega_{nm}, \vec{k}) \overset{(2)}{T^{nm}}_{\mu}(-\omega_{nm}, -\vec{k}) \right\} \\ & \times \left\{ \delta(\omega_{nm} - k) + \delta(\omega_{nm} + k) \frac{i\pi}{k} + P \frac{1}{2k} \left(\frac{1}{\omega_{nm} - k} - \frac{1}{\omega_{nm} + k} \right) \right\} \end{aligned} \tag{80}$$

We recognize again the following terms:

(i) Term containing $\delta(k) + \delta(-k)$. The contribution of this term to the dk-integral vanishes.

(ii) The term $P1/k$: This term corresponds to vacuum polarization.

(iii) The term with $(i\pi/k)[\delta(\omega_{nm}-k)+\delta(\omega_{nm}+k)]$. This term gives the spontaneous emission or absorption from level n to m or vice versa.

(iv) The term with $P\frac{1}{2k}\left(\frac{1}{\omega_{nm}-k}-\frac{1}{\omega_{nm}+k}\right)$ gives the Lamb shift.

These are our formulas for the radiative processes of the two-fermion system[18]. In the limit they go over to the fixed center Coulomb problem on the one hand, and for free particles to perturbative QED results.

For identical particles and particle-antiparticle system like positronium we have to antisymmetrize our currents as discussed in Sec. VI. Thus the mutual interaction action has two terms. The first is the usual direct interaction term

$$\begin{aligned} W^{\text{int}}_{e^-e^+}(1) &= -e^2\int dxdy\bar{\Psi}_I(x)\overset{(1)}{\gamma^\mu}\psi_I(x)D(x-y)\bar{\psi}_{II}(x)\overset{(2)}{\gamma_\mu}\psi_{II}(y) \\ &= -e^2\int dxdy\bar{\phi}_I(x,y)\overset{(1)}{\gamma^\mu}D(x-y)\overset{(2)}{\gamma_\mu}\phi(x,y) \end{aligned} \tag{81}$$

corresponding to the potential

$$V = -e^2\overset{(1)}{\gamma^\mu}\frac{1}{r}\overset{(2)}{\gamma_\mu} \tag{82}$$

The second term is

$$\begin{aligned} W^{\text{int}}_{e^-e^+}(2) &= e^2\int dxdy\bar{\phi}(x,y)\overset{(1)}{\gamma^\mu}D(x-y)\overset{(2)}{\gamma_\mu}\phi(y,x) \\ &= \frac{e^2}{2\pi}\int d\vec{r}\bar{\phi}_E(p_0,-\vec{r})\overset{(1)}{\gamma^\mu}\frac{e^{i\vec{k}\cdot\vec{r}}}{(p_0+p_0')^2-\vec{k}^2}\overset{(2)}{\gamma_\mu}\phi_E(p_0,-\vec{r})\frac{d\vec{k}}{(2\pi)^3} \end{aligned} \tag{83}$$

where p_0 and p_0' are the initial and final state energies and E is total conserved center of mass energy of the whole system. In the positronium the relative momentum is approximately zero so that we can set

$$p_\mu \cong (m,\vec{o})$$

and the action becomes

$$W^{\text{int}}_{e^-e^+}(\text{annihilation}) \cong \frac{2\alpha}{\text{m}^2}\int d\vec{r}\bar{\phi}(\vec{r})\overset{(1)}{\gamma^\mu}\delta(\vec{r})\overset{(2)}{\gamma_\mu}\phi(-\vec{r}) \tag{84}$$

Now we show that this term gives correctly the annihilation contribution to the hyperfine splitting in the $n=1$ state of positronium, for example. The effective potential above (84), when inserted into our wave equations gives an energy shift only for the levels $j=l\cong 0$ and for $j-1=l\cong 0$.

$$\delta E(j=\ell=0)\cong\frac{m\alpha^4}{2n^3}$$

and

$$\delta E(j-1=\ell=0)\cong\frac{m\alpha^4}{4n^3}$$

The difference is the annihilation contribution in the hyperfinesplitting

$$\delta E_{HfS}(\text{annihilation}) \cong \frac{3m\alpha^4}{12n^3} \tag{85}$$

To this order it agrees with perturbative QED. It is however obtained here in first quantized QED with selffields.

VIII. FURTHER RESULTS

There are still discrepancies between theory and experiment in almost all tests of QED. The Table III summarizes all measured levels in positronium, muonium and Hydrogen, positronium lifetimes, the anomalous magnetic moments a_e and a_μ, and some theoretical values in parenthesis. In reviewing some of these discrepancies, W-Lichten[18] writes "It seems likely that the problem lies in the difficulty of QED calculations which have not been carried out to a high order enough, perhaps a totally new type of calculation is needed". The self field approach to QED provides a new type of calculation. It's important to have a complementary or alternate method to perturbative QED, for a theory is tested not only against experiment but also against other theories in order to clarify the basic assumptions and concepts, specially in view of recent results that perturbative QED might be inconsistent or a trivial theory. Selffield QED modifies our notion of the quantized radiation field and the interpretation of quantum theory. The emphasis is shifted from the field to the source of radiation, an electronic charge distribution which objectively and deterministically evolves as a classical field and produces a selffield which acts back on the charge itself. Quantized properties of the light reflect the discrete frequencies of the oscillating charge distribution.

The two-body relativistic equation discussed in Sections V and VII gives us a possibility to make improved calculations for positronium and muonium, in particular. In positronium, the experiments seem to be more accurate than the theory and the perturbative calculations remain incomplete[19]. Considerable analytical work has been done on the study of the two-body equation (39)ff: separation of radial and angular parts and further reduction of the radial equations[20]. It turns out that the two-body equation, when the electromagnetic potentials are kept to order α^4, is exactly soluble with an energy spectrum

$$E^2 = \frac{M^2 + \Delta m^2}{2} + \frac{M^2 - \Delta m^2}{2}\left[1 + \frac{\alpha^2}{(n_r + \ell)^2}\right]^{-1/2} \tag{86}$$

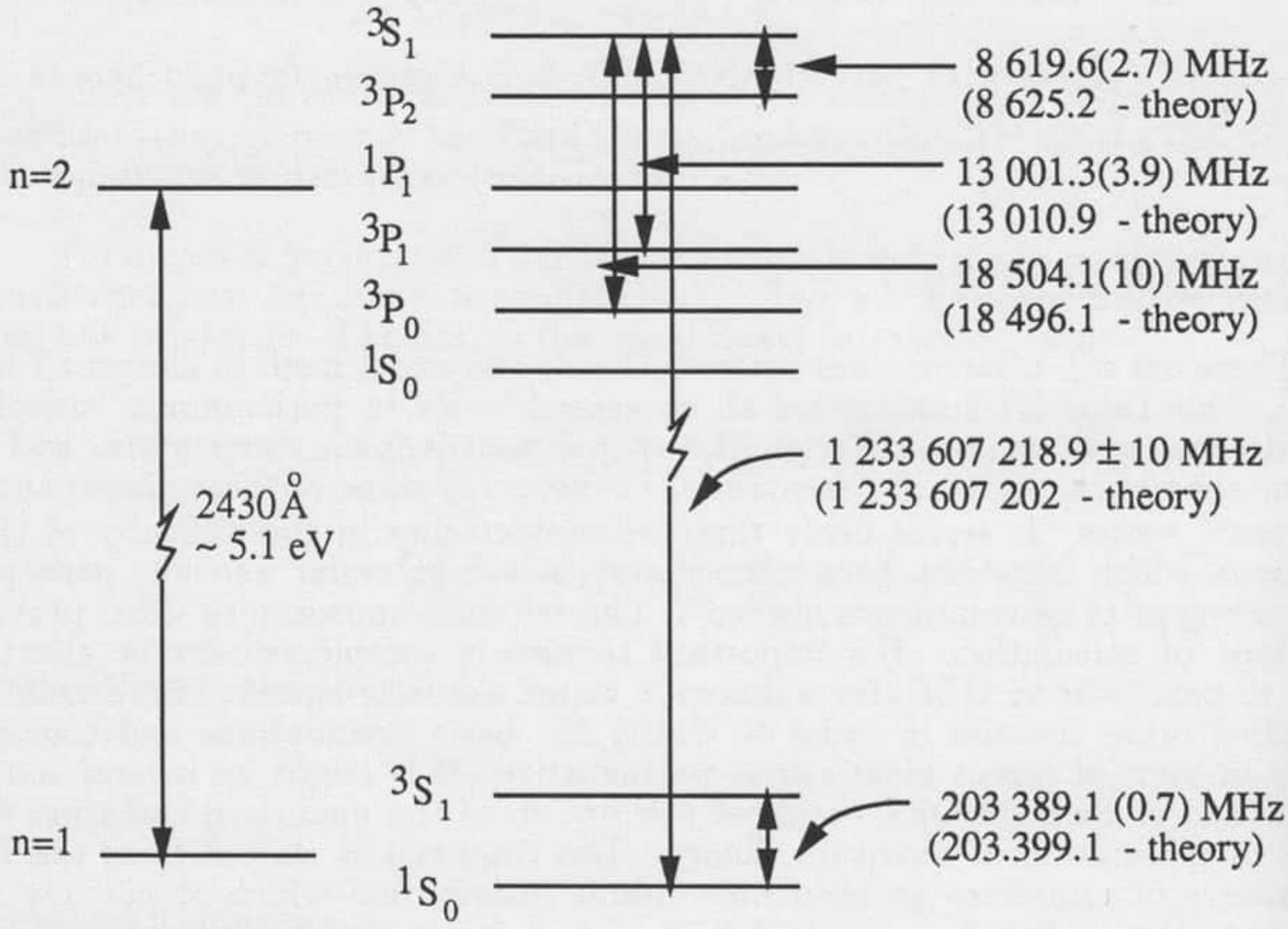
with $M = m_1 + m_2$, $\Delta m = m_1 - m_2$, generalizing the Dirac spectrum. We have treated the remaining potentials of order α^5 and higher as perturbations. But having tested the equation in this way, one can now make direct nonperturbative numerical calculations. The treatment of the negative energy states and the covariance of the equation has also been discussed[22] according to the methods outlined in Section VI. We give here some of the results[20].

1) For parapositronium, eq. (86), is exact including terms of the order α^4 since normal and anomalous magnetic moment terms do not contribute to this order. It gives

$$E^{\text{para ps}} = sm - \frac{m\alpha^2}{4n^2} - \frac{m\alpha^4}{2n^3(2j+1)} + \frac{11}{64}\frac{m\alpha^4}{n^4} + 0(\alpha^6) \tag{87}$$

TABLE III. BOUND STATE TESTS OF QED

POSITRONIUM

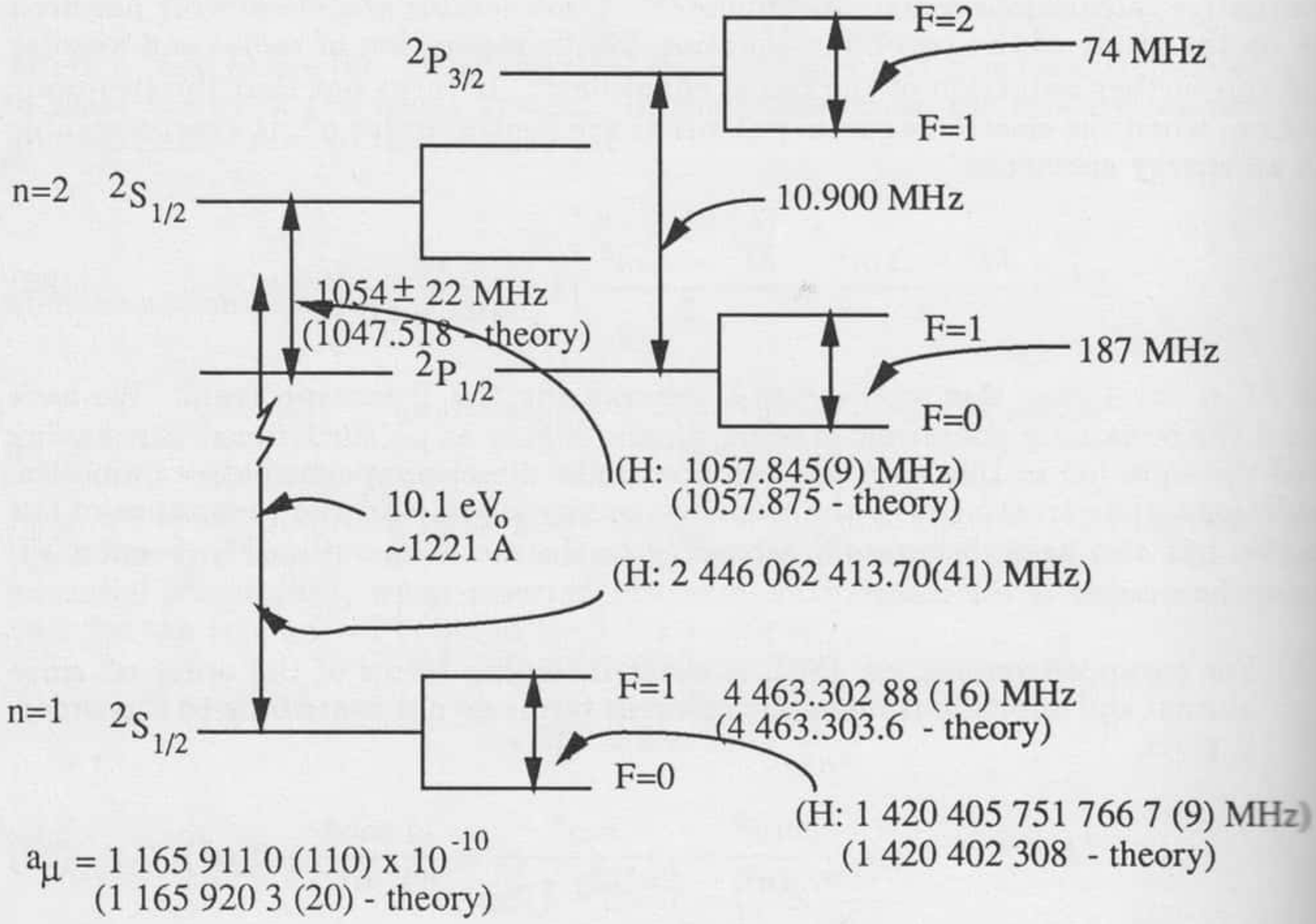


$\tau\ ^3S_1 \rightarrow 3\gamma = 7.0514\ \mu s^{-1}$
(7.0383 - theory)

$\tau\ ^1S_0 \rightarrow 2\gamma = 7.994\ ns^{-1}$
(7.9866 - theory)

$a_e = 1\ 159\ 652\ 188.4\ (4.3) \times 10^{-12}$
(1 159 652 192 (108)) - theory)

MUONIUM (AND H)



$a_\mu = 1\ 165\ 911\ 0\ (110) \times 10^{-10}$
(1 165 920 3 (20) - theory)

2) Introducing the anomalous magnetic moments a_1, a_2, which are in the selfenergy term, as a Pauli-coupling we obtain the ground-state hyperfine splitting

$$\Delta E^{\text{Hfs}} = \frac{8}{3}\frac{\zeta}{(1+\zeta)^2} m\alpha^4 \left[(1+a_1)(1+a_2) - \frac{3}{4}\zeta a_2^2 + \frac{3}{4}a_1 \frac{a_2}{(1+\zeta)^2}\right]$$

with $\zeta = m_1/m_2$. Numerically this gives 1420.348 MHz for H, and 4.463.060 MHz for muonium, compared to the experimental values 1420.405752 and 4.463302, respectively.

3) Positronium hypofinesplitting including the annihilation term, eq. (85), gives

$$\Delta E^{\text{Hfs}} = \frac{7}{12} m\alpha^4 + \frac{5}{12} m\alpha^4 \left(\frac{\alpha}{2\pi}\right).$$

This "Lambshift" term $-\frac{\alpha}{2\pi}(\frac{16}{9} + \ell n 2) m\alpha^4$ has to be added perturbatively, but we hope to calculate these terms and more eventually numerically.

4) Positronium $(n = 2, n = 1)$ splitting, including annihilation and anomalous magnetic moment contributions

$$\Delta E_{21} = \frac{3}{8} Ry - 0.468098\alpha^2 Ry - \frac{\alpha^2 Ry}{2\pi}\frac{35}{96}.$$

5) Positronioum fine structure

$$\Delta E(2^3S_1 - 2^3P_2) = -\frac{1}{12}\alpha^2 Ry + \frac{7}{48}\alpha^2 Ry - \frac{7}{480}\alpha^2 Ry + 0(\alpha^5)$$

$$\text{(recoil)} \quad \text{(annihilation)} \quad \left(\begin{smallmatrix}\text{normal magnetic}\\ \text{moment}\end{smallmatrix}\right)$$

$$= \frac{23}{480}\alpha^2 Ry + 0(\alpha^5)$$

6) H or muonium $(n = 2,\ n = 1)$ splitting

$$\Delta E_{21} = \frac{3}{8}\mu\alpha^2 + \frac{8}{128}\mu\alpha^4 + \frac{15}{128}\mu\alpha^4\frac{\zeta}{1+\zeta}$$
$$- \frac{7}{16}\mu\alpha^4\left(1 + \frac{2(a_1+\zeta^2 a_2)}{(1+\zeta)^2} - 2a_1a_2\frac{\zeta}{1+\zeta}\right)$$
$$- \frac{7}{12}\frac{\mu^2}{M}\alpha^4(1 + a_1 + a_2 + a_1a_2)$$

where $M = m_1 + m_2$, $\zeta = m_1/m_2$, $\mu = \frac{m_1 m_2}{M}$.

For other details and applications of self-field QED we refer to the literature listed in the Appendix.

References

1. See e.g. W. Thirring, *Principles of Quantumelectrodynamics* (Academic Press, N.Y. 1958).
   J.D. Bjorken and S.D. Drell, *Quantum Fields* (McGraw Hill, N.Y. 1965).
   R.P. Feynman, *Quantum electrodynamics* (Benjamin, N.Y. 1961).

2. A.O. Barut, in *Quantumelectrodynamics*, ed. P. Urban, Suppl. Acta Physica Austriaca, Vol. 2 (Springer, Berlin 1965); and *The Theory of the Scattering Matrix*, (Macmillan, N.Y. 1962), Ch. 13.

3. G. Scharf, *Finite Quantumelectrodynamics*, Springer 1989, and These Proceedings.

4. See e.g. C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw Hill, N.Y. 1964).

5. A.O. Barut and I.H. Duru, *Phys. Reports*, **172**, 1 (1989).

6. J. Schwinger, *Particles, Sources and Fields*, Vol. II (Addison Wesley, Reading, MA 1973).

7. H.A. Lorentz, *The Theory of Electrons*, (Dover, N.Y. 1952).

8. A.O. Barut, *Phys. Rev.*, **D10**, 3335 (1974).

9. A.O. Barut and N. Ünal, *Phys. Rev.*, **A40**, 5404 (1989).

10. A. Lees, *Philos. Mag.*, **28**, 385 (1939).

11. D.A.M. Dirac, *Proc. Roy. Soc.*, (London), **A268**, 57 (1962).

12. A.O. Barut and Y. Salamin, *Phys. Rev.*, **A37**, 2284 (1988).

13. A.O. Barut and Y. Salamin, Quantum Optics (in press).

14. A.O. Barut and N. Ünal, A Regularized Analytic Evaluation of Vacuum polarization in Coulomb field (to be published), and see also N. Ünal, These Proceedings.

15. A.O. Barut, J. Kraus, Y. Salamin and N. Ünal (to be published).

16. For more details on $(g-2)$ calculations see A.O. Barut and J.P. Dowling, *Zeits. f. Naturf.*, **44a**, 1051 (1989) and A.O. Barut, J.P. Dowling and J.F. van Huele, *Phys. Rev.*, **A38**, 4405 (1988).

17. For more details see A.O. Barut and N. Ünal, Complete QED of the electron-positron system and positronium (to be published).

18. W. Lichten, in *The Hydrogen Atom*, edit. by G.F. Bassani *et al* (Springer-1989), p. 39.

19. A. Rich *et al*, These proceedings.

20. A.O. Barut and N. Ünal, *J. Math. Phys.*, **27**, 3055 (1986), *Physica*, **142A**, 457 and 488 (1987).

21. A.O. Barut, *Physica Scripta*, **36**, 493 (1987), and in *"Constraint Theory and Relativistic Dynamics"*, L. Lusanna *et al*, edit., (World Scientific, 1987), p. 122.

Appendix: References on Selffield QED

1. Nonperturbative QED: The Lamb shift, A.O. Barut and J. Kraus, *Found. of Physics*, **13**, 189 (1983).

2. QED based on selfenergy, A.O.Barut and J.F. van Huele, *Phys. Rev. A*, **32**, 3887 (1985)

3. QED based on selfenergy vs. quantization of fields: Illustration by a simple model. A.O. Barut, *Phys. Rev. A*, **34**, 3502 (1986)

4. An exactly soluble relativistic quantum two fermion problem. A.O. Barut and N. Ünal, *J. Math. Phys.*, **27**, 3055 (1986)

5. A new approach to bound state QED. I. Theory, *Physica Scripta*, **142A**, 457 (1987), II. Spectra of positronium, muonium and Hydrogen. A.O. Barut and N.Ünal, *Physica*, **142A**, 488 (1987)

6. An approach to finite non-perturbative QED. A.O. Barut in "Proc. 2nd Intern. Symposium on Foundations of Quantum Mech.", *Phys. Soc. of Japan*, 1986, p. 323.

7. On the treatment of Møller and Breit potentials and the covariant 2-body equation for positronium and muonium, A.O. Barut, *Physica Scripta*, **36**, 493 (1987)

8. On the covariance of two-fermion equation, A.O. Barut, in "Constraint theory and Relativistic dynamics", (L. Lusanna *et al*, edit.) World Scientific, 1987; p. 122

9. Formulation of nonperturbative QED as a nonlinear first quantized classical field theory, A.O. Barut in "Differential Geometric Methods in Theoretical Physics", (H. Doebner *et al*, edit's), World Scientific 1987; p. 51

10. QED based on selfenergy: Spontaneous emission in cavities, A.O. Barut and J.P. Dowling, *Phys. Rev. A*, **36**, 649 (1987)

11. QED based on self energy: The Lamb shift and long-range Casimir-Polder forces near boundaries, A.O. Barut and J.P. Dowling, *Phys. Rev. A*, **36**, 2550 (1987)

12. QED based on selfenergy, A.O. Barut, *Physica Scripta*, **T21**, 18 (1988)

13. Relativistic spontaneous emission, A.O. Barut and Y. Salamin, *Phys. Rev. A*, **37**, 2284 (1988)

14. QED based on selfenergy: A nonrelativistic calculation of $(g-2)$, A.O. Barut, J.P. Dowling and J.F. van Huele, *Phys. Rev. A*, **38**, 4405 (1988)

15. QED based on selfenergy: Cavity dependent contributions of $(g-2)$, A.O. Barut and J.P. Dowling, *Phys. Rev. A*, **38**, 2796 (1989)

16. QED based on self fields, A Relativistic calculation of $(g-2)$, A.O. Barut and J.P. Dowling, *Zeits. f. Naturf*, **44A**, 1051 (1989)

17. Path integral formulation of QED from classical particle trajectories, A.O. Barut and I.H. Duru, *Phys. Reports*, **172**, 1–32 (1989)

18. Problème relativiste á deux corps en électyrodynamique quantique, *Heelv. Phys. Acta*, **62**, 436 (1989)

19. Selffield QED: The two-level atom, A.O. Barut and J.P. Dowling, *Phys. Rev. A*, (1 March 1990 issue)

20. QED based on selffields: On the origin of thermal radiation detected by an accelerated observer, A.O. Barut and J.P. Dowling, *Phys. Rev. A*, (March 1990)

21. Relativistic Spontaneous emission in Heisenberg representation. A.O. Barut and Y. Salamin, Quantum Optics, (1990)

22. Relativistic $2S_{1/2} \rightarrow 1S_{1/2} + \gamma$ decay rates of H-like atoms for all Z, A.O. Barut and Y. Salamin, *Phys. Rev. A*,

23. QED-The unfinished business, A.O. Barut in "Proc. III. Conf. Math. Physics" (World Scientific, 1990)

24. A regularized analytic evaluation of vacuum polarization in Coulomb field, A.O. Barut and Ünal, *Phys. Rev. D*,

25. The Foundations of Self-field Quantumelectrodynamics, A.O. Barut in "New Frontiers of Quantumelectrodynamics and Quantumoptics", (edit. by A.O. Barut), Plenum Press, NY 1990

26. QED Based on Selffields: Cavity effects, J.P. Dowling, *ibid*,

27. Complete QED of the electron-positron system and positronium, A.O. Barut and N. Ünal,