FUNDAMENTAL SYMMETRIES AND QUANTUM ELECTRODYNAMICS

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ABSTRACT

An essay on the symmetries of the Maxwell-Dirac system and on all the symmetries that follow from it.

I. The Importance of Electromagnetism

It can be said that perhaps 99.9% of all everyday physical phenomena concerning the constitution of matter and radiation in physics, chemistry and biology, can in principle be explained by a single simple theoretical system, namely the coupled Maxwell-Dirac equations (or their limiting cases, the Maxwell-Schrödinger, or the Maxwell-Lorentz equations). This is a tremendous achievement of the last hundred years of theoretical physics. This system describes phenomena at distances from about 10^{-16} cm (electron positron scattering at high energies) to radar probing of astronomical distances and energies from 10^{-5} eν (Lamb shift) to 40^{12} eν (e^+e^- accelerators).

The remaining .1% of the phenomena (nature leaves always a small door open to deeper levels) concern the rare events in radioactivity and cosmic rays, which led to the introduction of new particles, at first seemingly outside the Maxwell-Dirac system, beginning in 1932, namely the neutron and the neutrino. With this date the particle physics begins and with it we have the new phenomenon of forceful production in the laboratory of all sorts of new particles, some of which already occur in cosmic rays, some perhaps produced for the first time by man. Gravitation presumably plays little role in the formation of nuclei, atoms and molecules, but is, of course, dominant in the formation of celestial bodies.

Because the electromagnetic theory, and its extension after Maxwell-Lorentz, to include positron, electron spin and the wave properties of the electron, hence quantumelectrodynamics, works so well and so universally, we must first see if it can also account for these remaining phenomena of radioactivity, the properties of the nucleus and its disintegration products, before introducing completely new theories, new forces and new particles. This expectation is based on the realization that, as we shall see it in more detail, quantumelectrodynamics that we practice it today is in fact incomplete and does not give us the full information about the electromagnetic behavior at short distances or high energies, in fact an unknown territory--terra incognita--that must be explored.

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We have divided physical phenomena according to their energetic appearances into weak, strong, electromagnetic and gravitational. They also differ in their range (short-ranged or long-ranged interactions) or in being microscopic or macroscopic. However, electromagnetism can and does manifest itself in widely different strengths. For example, the "chemical force" between two neutral atoms is very weak compared to Coulomb force, is charge independent, hence seemingly non-electromagnetic, but has revealed itself, after quantum mechanics, to be a residual weak manifestation of electromagnetism when atomic structure is taken into account. This small residual force is of course of vital importance for the whole of chemistry and biophysics. Another example is the α-radioactivity, which is also very weak in general, and has also revealed itself not as a new force, but a largely electromagnetic and quantum phenomenon due to the tunneling of the α-particle through the electromagnetic barrier of the α-nucleus system. In both these instances we can describe the process as a new interaction vertex with appropriate new coupling constants: For α-decay, the interaction $g_\alpha \bar{\psi}_A \psi_B \phi_\alpha$ representing the vertex $A \longrightarrow B + \alpha$. For chemical force the interaction $g_{chem}\, \bar{\psi}_1 \bar{\psi}_2 \psi_3 \psi_4$ representing the reaction $H + H \longrightarrow H + H$, for example.

These are examples of a true unification in which the new coupling constants g_α, g_{chem} are completely eliminated in terms of the underlying electromagnetic coupling constant e. We could have put these interactions together with the electromagnetism into a larger system, and construct a gauge field theory with a broken symmetry. There is in fact a gauge theory of chemical forces plus electromagnetic forces. Instead g_{chem} and g_α are calculated in terms of e. Coming back to the four fundamental interactions of physics, weak, electromagnetic, strong and gravitational, the unification attempts of the recent decades have concentrated on putting all these separate forces side by side into a larger system and attribute the differences to symmetry breaking. The alternative that we wish to explore is to derive and calculate these interactions from electromagnetism which would be an already unified theory with possibly a single coupling constant e. We should remember that Newton's unification of terrestrial and celestial gravity, and Maxwell's unification of electricity and magnetism have a single coupling constant; the magnetic coupling constant is calculated in terms of e and c, $g_{magn} = e/c$, and c is known by independent measurements. These are true unifications.

With these remarks on the fundamental significance and possible universal role of electromagnetism I shall discuss the following topics in symmetry:

1) Symmetries of and from the Maxwell-Dirac System
2) Symmetries of electromagnetic matter: Two or many body systems that bind electromagnetically, and lead eventually to macroscopic symmetries
3) Symmetries of particle physics. The extrapolation of QED to short distances and a possible phase transition of QED at short distances to strong interactions.
4) Symmetries of the electron itself, the most basic of all particles.

II. Symmetries We Have Learned from Electrodynamics

Space-Time Symmetries. Maxwell's equations gave us the notion of the invariance under Lorentz transformations (Waldemer Voigt (1887) and Hendrik A. Lorentz), this in turn the relativistic particle equations (H. Poincaré (1904), and finally the physical interpretation of simultaneity and a new definition of inertial frames (A. Einstain (1905)). Thus the special relativistic symmetries of space-time originate from electrodynamics.

The electromagnetic field concept is in fact intimately related with the structure of space and time. The electromagnetic fields were originally thought to be, like in any other wave phenomena, waves in a medium, in an aether which fills and defines the whole space. This cannot be a rigid aether (Michelson-Morley experiment), but a deformable aether is perfectly relativistic. After the special relativity, we have gotten used to talk about waves without a medium (like ocean waves without the ocean) at the expense of introducing new physical quantities, e.g. potentials $A_\mu(x)$, whereas in a medium $A_\mu(x)$ would be simply the displacement of the aether from its equilibrium position. Either way, it is the wave operator, $\Box$, that makes the space-time of particle physics as we know it now.

Discrete Symmetries. The wave equation, or more generally the Maxwell's equations define also invariance under space-reflection (Parity P), and invariance under time reflection (time reversal T). There is one other important discrete symmetry, particle-antiparticle conjugation C which comes from the properties of the Dirac current $j^\mu(x)$ on the right hand side of the Maxwell's equations:

$$F^{\mu\nu},_\nu = - j^\mu \equiv - e \bar{\psi} \gamma^\mu \psi$$

$$*F^{\mu\nu},_\nu = 0 \qquad (1)$$

Without introducing any new coupling constants we can work with an electric charge - magnetic charge symmetry by replacing the second equation in (1) by

$$*F^{\mu\nu},_\nu = - k^\mu \qquad (2)$$

where k^μ is the current of magnetic monopoles. The coupling constant g in k^μ is determined by e by the charge quantization relation $eg = n/2$; $n = 0, 1, 2 \ldots$. Since no free magnetic monopoles have been discovered, we work usually in the sector $n = 0$.

Currents. The left hand side of Maxwell's equations have not changed since Maxwell even in quantum electrogynamics. The right hand side, the form of the current j^μ, describing the matter, has undergone considerable change, however, from the macroscopic currents of Maxwell, to the current of classical point charges of Lorentz, to the distributed currents of Schrödinger, and finally to the currents of spinning electrons of Dirac. The Dirac current describes both electrons and positrons, and to every motion of an electron there is another symmetric motion of the positron (particle-antiparticle symmetry C).

The electrodynamics gives us thus the full relativistic invariance (Poincaré symmetry with space and time reflections, P and T) as well as particle-antiparticle conjugation C due to the properties of the electronic current.

There is no indication anywhere else that the proper relativistic invariance is broken. As to the discrete symmetries P, C and T their apparent violation in some processes is best understood from the peculiar structure of the underlying particles, like neutrino and K_0- mesons.

Conformal Symmetry. Electrodynamics gave us also the conformal symmetry, a symmetry which contains in addition to the Poincaré Symmetry of Lorentz transformations and translations, also the dilations and inversions of coordinates. This 15-parameter symmetry group is first a property of the free electromagnetic field, but can be expanded to full electromagnetism if the current j_μ has special transformation properties.

The additional symmetry operations can be interpreted as the change of scales from point to point.

Self Consistent Coupling Between Field and Matter. Eq's. (1) are only one-half of electrodynamics. The other half describes the dynamics of the source-current, the Dirac equation

$$\{\gamma^\mu(i\partial_\mu - eA_\mu) - m\}\,\psi = 0\ , \tag{3}$$

or, a classical equation for particles, if the source is a point charge.

As H.A. Lorentz taught us, the electromagnetic field $F_{\mu\nu}$ is produced by the source, which in turn is driven by the field, hence both must be treated self-consistently. The way to guarantee this self-consistency is to derive both equations (1) and (3) from a single variational principle: The action of electrodynamics is

$$W = \int \{\bar\psi(\gamma^\mu i\partial_\mu - m)\psi - ej^\mu A_\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\}\, d^4x \tag{4}$$

where the first term is the kinetic energy of the source, the last the kinetic energy of the field, and the middle term the interaction between the field and matter. According to Leibnitz we live in the best of all possible worlds, and the extremum of the action W gives us not only both equations (1) and (3), but also the conservation laws and symmetries of electrodynamics that we talked about, and the consistency of these conservation laws with the time-evolution of the system.

III. Symmetries of the Electromagnetic Matter

Electromagnetic matter consists of the bound states and other clusters of basic particles formed solely by their electromagnetic interactions. Thus starting with electrons, muons and protons we have the formation of positronium, muonium, H-atoms, as well as, of course, other atoms, molecules, biological molecules, up to crystals and condensed matter. The structure of proton and nuclei plays a very little role (e.g., hyperfind structure).

I shall first show how the symmetry and dynamical groups of the two-body problem follows from the basic action of electrodynamics, and how we obtain in the limit the dynamical groups of the one-body problem in a potential--a problem that has been widely studied since the 1960s. In particular, the postulated infinite-component wave equations can now be given a field theoretic derivation based on electrodynamics. In order to derive the equations and symmetries of the two-body system from field theory we consider two matter fields ψ_1 and ψ_2 as the source of the current and the action (4) becomes

$$\begin{aligned} W = \int dx[&\bar\psi_1(\gamma^\mu i\partial_\mu - m)\psi_1 + \bar\psi_2(\gamma^\mu i\partial_\mu - m_2)\psi_2 \\ &-(e_1\bar\psi_1\gamma^\mu\psi_1 + e_2\bar\psi_2\gamma^\mu\psi_2)A_\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}] \end{aligned} \tag{5}$$

If we choose a gauge such that A^μ, $\mu = 0$, then the first Maxwell eq. (1) becomes $\Box A^\mu = j_\mu$ and can be solved for A_μ

$$\Box A_\mu = \int dy\, D(x-y)\,[e_1\bar\psi_1\gamma_\mu\psi_1 + e_2\bar\psi_2\gamma_\mu\psi_2] \tag{6}$$

where $D(x-y)$ is the Green's function of the wave operator $\Box$. The potential A_μ in (6) -- it is the Lienard-Wiechert potential--can be called the

self-field of the electron: The Dirac current, $e\bar{\psi}\gamma^{\mu}\psi$, is assumed to produce a field, like any other current, which acts not only on the other particles, but also on itself.

Inserting A_{μ} of (6) into W (also in the term $F_{\mu\nu}F^{\mu\nu}$) we obtain a total interaction action (kinetic parts being unchanged)

$$W^{int} = \int dxdy\ [j_{1\mu}(x) + j_{2\mu}(x)]D(x-y)[j_1^{\mu}(y)+j_2^{\mu}(y)] \tag{7}$$

representing two mutual interactions and two self-interactions of currents. There are two variational principles for our action (5)-(7).

(i) If we vary W with respect to ψ_1 and ψ_2 separately, we obtain coupled nonlinear Hartree-type equations. I call such a system a "separated two-body quantum system"

(ii) We can vary W with respect to the composite field

$$\Phi(x_1x_2) = \psi_1(x_1)\psi_2(x_2) \tag{8}$$

only. We now get a linear equation for Φ only (plus some extra-terms coming from self-interactions). This is the quantum mechanical two-body equation in configuration space, well-known in the standard nonrelativistic many-body problem. Once such a wave equation for $\Phi(x_1x_2)$ is written, the solution is no longer factorizable. I call such a system a "nonseparated quantum 2-body system." The peculiar long-range correlations of quantum 2-body systems (e.g. Einstein-Podolsky-Rosen problem) are due to this configuration space wave functions.

I think both types of systems, separated and nonseparated, occur in quantum systems. For example, in the H_2-molecules, the two electrons are not separated, their wave functions are in the tensor product space and must be antisymmetrized with output to the exchange of the two electrons; they mix. But the two protons are separated. We do not antisymmetrize the wave function with respect to the protons. The Born-Oppenheimer method treats protons and electrons on different footing. I think this is also physically so: an H_2-molecule is an individual single system defined by the positions of the nuclei; we may use the probabilities for the distribution of the two electrons.

The two-body relativistic equation for the composite field $\Phi(x_1x_2)$ is obtained as follows. We first express the mutual interaction action in (7) in terms of Φ:

$$W^{int} = \int dx_1dx_2\bar{\Phi}(x_1x_2)\gamma^{\mu}\circledast\gamma_{\mu}\Phi(x_1x_2)D(x_1x_2) \tag{9}$$

Φ is a 16-component spinor with two spinor indices $\Phi_{\alpha\alpha'}$, so this equation has to be understood in the tensor product space of two Dirac Spin algebra, i.e.

$$\bar{\Phi}_{\alpha\alpha'}\ \gamma^{\mu}_{\alpha\beta}\circledast\gamma_{\mu\alpha'\beta'}\ \Phi_{\beta\beta'}$$

The self-energy terms are a bit complicated; we shall indicate them separately at the end.

In order to write the two kinetic energy terms in eq. (5) in terms of Φ, we multiply the first term in W with the normalization integral $\int d\vec{x}_2\ \bar{\psi}_2\ \gamma\cdot n\psi_2$ and the second term with the integral $\int d\vec{x}_1\ \bar{\psi}_1\ \gamma\cdot n\ \psi_1$. In this way also the kinetic energy terms can be written in terms of

Φ and $\bar{\Phi}$. We now vary the action with respect to $\bar{\Phi}$ and obtain the two-body equation:

$$\{(\gamma^\mu i\partial_\mu - m_1) \circledast \gamma\cdot n + \gamma\cdot n \circledast (\gamma^\mu i\partial_\mu - m_2) - e_1 e_2 \frac{\gamma^\mu \times \gamma_\mu}{r_\perp} + V_{self}\} \Phi = 0 \quad (10)$$

Here we always write in the tensor product the spin matrices and indices of the particle 1 first, those of the second particle second, e.g. $\gamma_\mu \times \gamma^\mu$. Further, n^μ is a time-like four-vector and $r_\perp = [((x_1-x_2)\cdot n)^2 - (x_1-x_2)^2]$ is the relativistic distance. We can choose $n^\mu = (1000)$, then $r_\perp = r$. Self energy terms V_{self} we shall explain later. Equation (10) is fully covariant, and more importantly contains a single time. In order to see this we introduce center of mass and relative coordinates by the usual transformations.

$$P = p_1+p_2, \quad r=x_1-x_2, \quad p=(1-a)p_1-ap_2, \quad R = ax_1+(1-a)x_2$$

and obtain the equation

$$\{\Gamma^\mu P_\mu - g^\mu p_\mu - m_1 1\odot\gamma\cdot n - m_2 \gamma\cdot n\odot 1 - e_1 e_2 \frac{\gamma^\mu \times \gamma^\mu}{r} + V_{self}\} \Phi = 0 \quad (11)$$

where $\Gamma^\mu = a\gamma^\mu \circledast \gamma\cdot n + (1-a)\gamma\cdot n \circledast \gamma^\mu$

$$g^\mu = \gamma^\mu \circledast \gamma\cdot n - \gamma\cdot n \circledast \gamma^\mu$$

Now we see indeed that component of g^μ parallel to n^μ vanishes identically. In fact separating all four-vectors into a component parallel to n^μ and another perpendicular to n^μ, e.g.

$$\Gamma^\mu_{\parallel} = (\gamma\cdot n)n^\mu, \quad \gamma^\mu_\perp = \gamma^\mu - (\gamma\cdot n)n^\mu \quad ,$$

we obtain

$$\{P_{\parallel} - \alpha_\perp\cdot P_\perp - g_\perp\cdot p_\perp - m_1\gamma\cdot n \circledast I - m_2 I \circledast \gamma\cdot n - e_1e_2 \frac{1}{r_\perp}(1-\alpha_{1\perp}^\mu\cdot\alpha_{2\perp}^\mu)$$

$$+ \tilde{V}_{self}\} \Phi = 0 \quad (12)$$

$P_{\parallel}$ is the Hamiltonian

$$\alpha_\perp^\mu = a\gamma\cdot n\gamma^\mu \circledast I + (1-a)\, I \circledast \gamma_\perp^\mu\gamma\cdot n$$

$$g_\perp^\mu = \gamma\cdot n\,\gamma_\perp^\mu \circledast I - I \circledast \gamma_\perp^\mu\,\gamma\cdot n$$

Or, for $n^\mu = (1000)$,

$$\{P_0 - \vec{\alpha}\cdot\vec{P} - \vec{g}\cdot\vec{p} - m_1\beta_1 - m_2\beta_2 - \frac{e_1e_2}{r}(1-\vec{\alpha}_1\cdot\vec{\alpha}_2) + \tilde{V}_{self}\} \Phi = 0$$

$$\vec{\alpha} = a\vec{\alpha}_1 + (1-a)\vec{\alpha}_2, \quad \vec{g} = \vec{\alpha}_1 - \vec{\alpha}_2 \quad (13)$$

For Coulomb problem the operator

$$\Theta = r\, P_0$$

has a simpler spectrum than $P_0 = H$ itself:

$$\Theta = r\vec{\alpha}\cdot\vec{P} + \vec{g}\cdot r\vec{p} + (m_1\beta_1 + m_1\beta_2)r + e_1e_2(1-\vec{\alpha}_1\cdot\vec{\alpha}_2) + r\tilde{V}_5 \quad (15)$$

Dynamical Symmetries. For a fixed center of mass momentum $\underline{P}$(e.g. $\underline{P} = 0$), the operators κ, κp are in the Lie algebra of SO(4,2), the well-known dynamical group of the H-atom without spins. The coefficients $\alpha\cdot\underline{P}$, g, $(m_1\beta_1 + m_2\beta_2)$, $\alpha_1 \cdot \alpha_2$, . . . are in the Lie algebra of the tensor product of two Dirac representations of SO(4,2). Thus, the full dynamical group of the two-body problem with spins is, as expected, $SO(4;2)_{orbital}$ ⓧ $SO(4,2)_{Dirac}$ ⓧ $SO(4,2)_{Dirac}$.

Neglecting self-energy terms V_{self} which for atoms contains small terms like Lamb-shift and spontaneous emission, we can pass easily to the limits of first to one-body Dirac equation, then to spinless case, and finally to nonrelativistic Kepler problems.

If one of the particles is heavy, $m_2 \to \infty$, $\overset{(2)}{\gamma}_0 \sim 1$, $\overset{(2)}{\vec{\gamma}} \to 0$, $a = m_1/M \to 0$, the Hamiltonian of the first particle in the center of mass frame with $P_0 = p_0{}^{(1)} + m_2$

$$\mathcal{H}^{(1)} = \kappa P_0{}^{(1)} = \vec{\alpha}\cdot\kappa\vec{p}_1 + \kappa\beta m_1 + e_1e_2 \tag{15}$$

i.e. the one-body Dirac Hamiltonian.

Furthermore, eq. (11) written concisely as

$$\{\Gamma^\mu P_\mu + K\}\,\Phi = 0 \tag{16}$$

where K is a matrix, is an infinite-component wave equation which, as is also well-known, describe composite systems realistically.

To summarize, the dynamical group approach to quantum systems, in particular the infinite component wave equation, can be derived from first principles from an underlying electron dynamics field theory. In this way the parameters of the infinite component wave equations are determined in terms of the masses and coupling constants of the basic constituents.

Finally we remark that the full dynamical algebra of our system (13) or (16), when the generators of the Poincaré group $P_0, \vec{P}_1$ . . . are included, will be an infinite-dimensional algebra of the Kac-Moody type. This is because, for a composite system we have a highly reducible representation of the Poincaré group representing the infinitely many mass and spin states, and the generators of the dynamical group connect these different mass states.

From Microscopic Symmetry to Macroscopic Symmetry

The previous method of deriving equations for 2-body system from field theory can be extended to 3 or more particles. But which many body systems do actually lead to stable (or nearly stable) bound states is a question of the sizes of the parameters; the stability of atoms depends on the nuclear charge Z, for example.

But a new situation occurs when we go from atoms to molecules. Consider the simplest molecule H_2 consisting of two hydrogen atoms. It does not seem to be possible to understand H_2 starting from a 4-body Schrödinger equation representing two protons and two electrons. Rather, as we have already mentioned, we have spontaneously broken the permutation symmetry for the two-protons, that is "separate" the two protons and apply quantum mechanics only to the electrons. The Born-Oppenheimer method fixes the positions of the protons, treats the two electrons quantum mechanically,

and then considers the small oscillations of the two protons separately. In doing this we take into account that a molecule is a definite single objective quantum system, and not a probability distribution. The individuality of quantum systems is established, I think, at this level. Another way of expressing the Born-Oppenheimer procedure is to say that the distance R between the protons obeys at first a superselection rule, i.e. it is not quantized. After solving the problem with fixed R we allow for oscillations of the protons around their center of mass but without mixing. In fact it is possible to treat the two-body problem in quantum mechanics without quantizing the center of mass: only relative coordinates are quantized; one obtains the same result as the usual theory. Super-selection rules are the proper way to deal with nonquantized dynamic variables.

Continuing further from molecules to more complex systems, I have discussed in "Symmetry in Science II" the question whether we can derive crystal symmetry from first principles, i.e. from an N-body Schrödinger equation for $\psi(x_1 \ldots x_N, R_1 \ldots R_m)$ for N electrons and M nuclei. The answer, I believe, is no. The positions of the nuclei, i.e. the crystal symmetry, h are determined by essentially classical equilibrium or group theoretical arguments. The large permutation and rotation symmetries of nuclei is broken down to a smaller crystal symmetry. But once crystal symmetry is given, we can then quantize the electrons in this given environment in which the electron clouds spread and mix.

These are, I think, limitations to the unquestioned use of quantum rules to the structure of matter.

IV. The Extrapolation of Electrodynamics to Short Distances

We have discussed the formation of two- or more-body bound states in electrodynamics corresponding to atomic and molecular structure. In these instances the dominant force is the mutual interaction between the particles; the self-fields give for these states only small corrections (Lamb shift, spontaneous emission). The size of the atomic structures are determined by the constants: m(mass of the electron), α(fine structure constant), c and $\hbar$, and nuclear charge z.

There is, however, a second regime where the self potentials dominate and the mutual forces between the particles are small corrections. To see this we go back to the general action (7). The self-interaction terms are:

$$W_{\text{int}}^{\text{self}} = e_1^2 \int dxdy\ \bar{\psi}_1(x)\gamma^\mu\psi_1(x)\ D(x-y)\bar{\psi}_1(y)\gamma_\mu\psi_1(y)$$
$$+ e_2^2 \int dxdy\ \bar{\psi}_2(x)\gamma^\mu\psi_2(x) D(x-y)\bar{\psi}_2(y)\gamma_\mu\psi_2(y) \qquad (17)$$

When we pass to the composite field $\Phi(x_1 x_1)$ defined by eq. (8), we can evaluate the self potential V_{self} in eq. (10) and rewrite eq. (10) now completely as

$$\{[\gamma^\mu(p_{1\mu} - e\, A_\mu^{(1)}) - m_1] \otimes \gamma\cdot n + \gamma\cdot n \otimes [\gamma^\mu(p_{2\mu} - e_2 A_\mu^{(2)}) - m_2]\}\, \Phi(x_1, x_2) = 0$$
$$A_\mu^{(1)} = \frac{1}{2} e_2 \frac{\gamma_\mu \gamma\cdot n}{r} + A_\mu^{(1)\,\text{self}}$$
$$A_\mu^{(2)} = \frac{1}{2} e_1 \frac{\gamma\cdot n \gamma_\mu}{r} + A_\mu^{(2)\,\text{self}} \qquad (18)$$

where the self-potentials are given by

$$A_\mu^{(1)\,self}(x_1) = \int dzdu\ \mathcal{D}(x_1-z)\bar{\Phi}(z,u)\gamma_\mu \otimes \gamma\cdot n\ \Phi(z,u)$$

$$A^{(2)\,self}(x_2) = \int dzdu\ \mathcal{D}(x_2-u)\bar{\Phi}(z,u)\gamma\cdot n \otimes \gamma_\mu \Phi(z,u)$$

In the center of mass frame, the Hamiltonian form of (18) is

$$\{\vec{\alpha}_1\cdot(\vec{p}_1-e_1\vec{A}_1)+\beta_1 m_1+\vec{\alpha}_2\cdot(\vec{p}_2-e_2\vec{A}_2)+\beta_2 m_2+e_1V_1+e_2V_2\}\Phi(x_1,x_2)$$
$$= E\Phi(x_1,x_2) \qquad (19)$$

where

$$\vec{A}_1 = \frac{e_2}{2}\frac{\vec{\alpha}_2}{\hbar} - \frac{e_1}{2}\int dzdu\ \mathcal{D}(x_1-z)\Phi^+(z,u)\vec{\alpha}_1\Phi(z,u)$$

$$\vec{A}_2 = \frac{e_1}{2}\frac{\vec{\alpha}_1}{\hbar} - \frac{e_2}{2}\int dzdu\ \mathcal{D}(x_2-z)\Phi^+(z,u)\vec{\alpha}_2\Phi(z,u)$$

$$V_1 = \frac{e_2}{2}\frac{1}{\hbar} - \frac{e_1}{2}\int dzdu\ \mathcal{D}(x_1-z)\Phi^+(z,u)\ \Phi\ (z,u)$$

$$V_2 = \frac{e_1}{2}\frac{1}{\hbar} - \frac{e_2}{2}\int dzdu\ \mathcal{D}(x_2-z)\Phi^+(z,u)\ \Phi\ (z,u)$$

Equation (18) is exact from the point of view of our classical relativistic self consistent field theory, and from the point of view of interpreting ψ as an objective representation of electronic matter. But equation (18)-(19) are rather complicated. We write it for the case when one of the particles is very heavy, as in H-atom, and its field is represented by a fixed external Coulomb potential.

$$[\gamma^\mu(i\partial_\mu - eA_\mu^{ext})-m]\ \psi = e^2\gamma^\mu\psi(x)\int dy\bar{\psi}(x)\gamma^\mu\psi(x)\ \mathcal{D}(x-y)\bar{\psi}(y)\gamma_\mu\psi(y) \qquad (20)$$

This is now a single nonlinear integrodifferential equation for a single particle in an external field. If the self-energy term on the right hand side of this equation is treated iteratively, around the stationary solutions of the external field problem, then it is possible to obtain all the radiative corrections of quantum electrodynamics, i.e.

(*i*) spontaneous emission
(*ii*) Lamb shift
(*iii*) vacuum polarization
(*iv*) anomalous magnetic moment,

without second quantization of fields. Thus quantized electromagnetic field on the one hand, and self-field on the other hand are two dual ways of dealing with radiative processes. But now we can extrapolate eq. (20) to short distances, whereas we cannot do this in the perturbative QED--it would mean to be able to sum infinitely many Feynman diagrams.

Nonlinear equations of the type (20) have another regime in which the nonlinearity dominates over the external field; we may get new type of solutions corresponding to a self-focusing, or self-organization of the system. Such localized solutions are known for equations having soliton solutions. It has then been conjectured that electrodynamics should exhibit a phase transition at short distances to new self-organized states. To be more specific the electron-positron system (e^+,e^-), for example, which we know to form the atomic positronium at large distances

due to mutual interaction, should also form new states at short distances due to their own self fields. Now one of the effects of self energy is to give to the particles an anomalous magnetic moment whose value depends on the external field itself, self-consistently. In many models with an anomalous magnetic moment it is possible, in fact to show that new states of (e^+e^-) occur at distances of the order of the classical electron radius (α/m), which is also the typical hadronic distance. The masses of such states are multiples of $2m/\alpha \sim 140$ MeV.

This basic idea of a phase transition in QED at short distances gives us a possibility to re-examine the new particles, heavy leptons, mesons and baryons, as new composite states of electromagnetism. Although the dynamics is difficult and not completely solved, it is possible to completely understand the kinematics, that is the classification of particles and their quantum numbers on the basis of two fundamental absolutely stable particles only, the electron and its neutrino. All other particles, according to this view, are composite and unstable--but two of them apparently with an extremely long life-time, proton and muon-neutrino.

V. The Symmetries of the Electron

Finally, I discuss some new results concerning the structure of the electron itself, the most fundamental of all particles (see Sect. IV). It is in the structure of the electron that we must look for the origin of the many rather mysterious qualities of the electron; the spin, the charge, Pauli exclusion principle, the existence of antiparticles, and the existence of its partners, electron neutrino on the one hand and heavy leptons (μ,τ) on the other hand. The electron, for all these properties, is more than just a point particle, or an irreducible representation of the Poincaré group. Most importantly, the symmetry between heavy leptons, that is the identical behavior of e, μ, τ leptons excepting their mass, seems to defy any explanation so far. This problem is known as the existence of three families of leptons each with their own neutrinos. In standard models all these leptons are assumed to be elementary. The structure of the electron and its self field may give us a possibility that these heavy leptons are in fact in some sense "excited states" of the electron itself.

An intuitive picture of the Dirac electron can be obtained by a classical model which gives us very naturally the origin of spin and antiparticles; it may also lead to "excited" states. This classical, but of course covariant, model is most simply described in terms of an invariant time parameter τ by two pairs of conjugate variables: (x_μ,p_μ), the usual coordinates and momenta, and the internal spin variables $(\bar{z}, z)$, where $z_\alpha(\tau)$ are classical 4-component spinors with $\bar{z} = z^+\gamma^0$ its injugate. The theory is defined by the action

$$W = \int d\tau (i\lambda \dot{\bar{z}} z + p_\mu \dot{x}^\mu - p_\mu \bar{z}\gamma^\mu z + eA_\mu \bar{z}\gamma^\mu z) \tag{21}$$

up to a total time derivative. It is thus formulated on a larger phase-space but it is a symplectic Hamiltonian system. The two fundamental constants are λ, with the dimension of action, and e $(c=1)$. The mass enters later as the value of the integral of motion $H = \bar{z}\gamma^\mu z(p_\mu - eA_\mu)$, the "Hamiltonian with respect to τ" (or the mass). When this theory is quantized--either canonically by replacing Poisson-brackets with commutators, or by a Schrödinger quantization, or by a path integral quantization--one obtains the Dirac equation. But the concepts of spin and antiparticles are already present in the theory (21) as follows. If we solve the equations of motion resulting from (21) for a free particle, we find

that the natural motion of the particle in space time is a helix, around a fictitious center of mass which moves linearly like a relativistic particle. The frequency and radius of the helical motion are $2m$ and $\frac{1}{2}m$, respectively. Now the spin of the particle turns out to be simply the orbital angular momentum of the helical motion with respect to the center of mass. Furthermore, particles and antiparticles correspond to the positive and negative frequencies of helical motion, or right and left helical motions.

We thus see that such a microscopic dynamical system as described by eq. (21) has many remarkable symmetry properties which are then transformed to complex systems that they form. We should like to emphasize that no "force" is necessary to keep the particle in a helical path. The existence of internal variables does it automatically. This is a beautiful example of Heinrich Hertz's "forceless mechanics," where a geometry in a higher dimensional space implies forces in ordinary space-time. Another way to put this is to say that electron has other coordinates than just position and momenta.

It remains to be seen whether the internal structure of the electron can give us a deeper understanding of the existence of the heavy leptons, and why every lepton comes with its own neutrino.

CONCLUSIONS

The electrodynamics has been, since its conception over a hundred years ago, a most enduring theory. It has unified an enormous range of phenomena under one simple set of laws. These are the hallmarks of true scientific knowledge: general validity, extreme simplicity, freedom from arbitrary parameters. Quantumelectrodynamics is rightly called the best theory that physicists have ever built. In contrast the current formulations of the physics of nuclei and particles have shown such as complexity, hundreds of fundamental objects, many new forces and models with dozens of parameters, that we are undoubtedly far from a basic understanding of these phenomena. It is generally believed that these phenomena have nothing to do with electromagnetism.

In this essay I have tried to show not only the central role of electromagnetism in our understanding of the structure of all ordinary matter, but also the exciting possibility that the behavior of electromagnetic interactions at short distances is very likely to be quite different than at large distances, and that they undergo a phase transition and might very well explain the occurrence of multiples of new particle states at high energies. Simplicity may again be restored as the most important feature of the scientific endeavour.

REFERENCES

More technical details of the topics discussed in this essay may be found in the following reviews and in references listed therein.

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