Quantum electrodynamics based on self-energy, without second quantization: The Lamb shift and long-range Casimir-Polder van der Waals forces near boundaries

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Using a previously formulated theory of quantum electrodynamics (QED) based on self-energy, we give a general method for computing the Lamb shift and related Casimir-Polder energies for a quantum system in the vicinity of perfectly conducting boundaries. Our results are exact and easily extendable to a full covariant relativistic form. As a particular example we apply the method to an atom near an infinite conducting plane, and we recover the standard QED results (which are known only in the dipole approximation) in a simple and straightforward manner. This is accomplished in the context of the new theory which is not second quantized and contains no vacuum fluctuations.

I. INTRODUCTION

Standard quantization of the free electromagnetic field predicts a zero-point energy of $\frac{1}{2}\hbar\omega$ per normal mode, a fact that is disconcerting as it leads to an unobserved infinite energy density for the vacuum. According to one school of thought the zero-point energy cannot by physically real, and so it is blatantly subtracted from the Hamiltonian in a transfinite rescaling of the energy. Bohr has pointed out repeatedly that this cannot be the whole story;[1] for in general relativity, energy density leads to space-time curvature through the Einstein field equations, and one cannot simply flatten out the metric by rescaling the energy axis.

Another school would like to keep the vacuum fluctuations and interpret them as physically real: with real consequences. Thus phenomena such as spontaneous emission and the Lamb shift are viewed by many physicists as being *caused* by the electron's interaction with real electromagnetic field fluctuations.[2-4] But to quote Jaynes,[5] "To those who believe that zero-point fluctuations are the physical cause of the main part of the Lamb shift, they must then be 'very real things' at least up to the Compton cutoff frequency, $\hbar\omega=mc^2$, to get the right Bethe logarithm. Please calculate the *numerical value* of the resulting energy density in space, the turbulent power flow from the Poynting vector, etc., and then tell us whether you still believe the zero-point fluctuations are physically real (for the Poynting vector, we get 6×10^{20} megawatts cm^{-2}; the total power output of the sun is about 2×10^{20} megawatts; real radiation of that intensity would do a little more than just shift the 2s level by four microvolts)." Thus not all physicists are comfortable with the fluctuation point of view. In fact, spontaneous emission and the Lamb shift can equally well be explained without recourse to a fluctuating vacuum in theories which incorporate the radiation reaction of the self-field of the particles.[5-9] There indeed seems to be a deep connection between the vacuum fluctuation and the self-field interpretations of spontaneous emission; as first pointed out by Callen and Welton[10] in their famous paper on the generalized fluctuation-dissipation theorem, and as later elaborated on by Senitzky[11] and Milonni *et al.*[12]

The situation is much the same when it comes to long-range retarded van der Waals forces. In 1948 Casimir and Polder showed that one could derive the retarded London–van der Waals interaction between an atom and a conducting plate, or between two neutral atoms, by considering their mutual coupling to the second-quantized electromagnetic zero-point fluctuations.[13] In this same year Casimir computed in a similar fashion the zero-point force between two parallel conducting plates;[14] a force that was later measured experimentally.[15,16] These results are often touted as proof of the reality of the vacuum fluctuations.

However, Lifshitz *et al.*,[17-19] Schwinger *et al.*,[20] and also Milonni[21] have all been able to derive these same results in the context of self-fields and source theory without any need for a zero-point energy of the vacuum. Milonni, in his partially second-quantized approach, illustrates once more that there is an underlying and complementary nature to the two concepts of self-fields and vacuum fluctuations. This perhaps tells us that, in truth, our theoretical predilection for treating the two phenomena separately can never in practice be realized—an electron cannot be isolated from its self-field, nor can it ever actually be uncoupled from the vacuum.

In the present paper we shall be concerned with the recent formulation of quantum electrodynamics (QED) as advanced by Barut and Kraus,[7] and Barut and van Huele[8] in which there is no second quantization, but rather the self-fields of the sources are included from the beginning. The authors show that it is then possible to eliminate the self-field from the coupled Maxwell-Dirac equations, leaving behind a nonlinear integrodifferential equation. This equation is solvable by an iterative procedure, and the correct Einstein A coefficient for spontaneous emission, as well as the Lamb shift are recovered.

We will now show how these previous results are modified in the presence of perfectly conducting boundaries. The discussion of spontaneous emission near a boundary has been given elsewhere;[22] here we will only consider the Lamb shift and the associated Casimir-Polder force. As an example of a particular boundary we consider an atom near an infinite conducting wall: obtaining the Casimir-Polder energy for such a configuration as well as a general formula for the boundary-induced shift to any energy level. Our method is simpler and more compact than the standard QED computations of these effects and illustrates that second quantization and the notion of zero-point fluctuations are not requisite for obtaining them. This procedure can also easily be extended to a full covariant form—although we restrict ourselves here to the nonrelativistic case in order to compare with experiment and previous work by other authors.

In order to compare with the literature then we consider the two limiting cases: (a) the atom close to the plate and (b) the atom far from the plate. (Our general formula, however, is good for any distance.) In each case we estimate the additional shift of the 2s level of hydrogen as induced by the plate. These shifts depend on the atom-wall separation, and, although very small for macroscopic distances, they could conceivably become observable considering recent advances in surface physics, such as the scanning tunneling microscope, where individual atoms at the tip of a needle are suspended a few angstroms above a crystal interface.[23] More importantly, boundary effects on the $g-2$ value of the electron as measured in Penning traps[24] could be computed by our method. Work is in progress on this aspect of the theory; an entirely new approach could throw some light on the controversy surrounding previous theoretical computations of this effect.[25–28]

II. THEORY

In the nonrelativistic version of quantum electrodynamics based on self-energy, Barut and van Huele[8] show that the Lamb shift can be derived from an action principle applied to a Schrödinger matter field ϕ coupled to a classical electromagnetic field, $A_\mu = A_\mu^{\text{self}} + A_\mu^{\text{external}}$. The field obeys Maxwell's equations in the form

$$\Box A_\mu^{\text{self}}(x) + A_{\nu,\mu}{}^{\nu}(x) = -j_\mu(x) , \tag{1}$$

where $j_\mu(x)$ is the usual electron probability four-current. This equation can be solved formally as

$$A_\mu^{\text{self}}(x) = \int dy^4 D_{\mu\nu}(x-y) j^\mu(y) \tag{2}$$

with $D_{\mu\nu}$ the standard electromagnetic field (free-space) Green's function, given in the Coulomb gauge as

$$D_{\mu\nu}(x) = -\frac{1}{(2\pi)^4}\int dk^4 \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{k_0^2 - |\mathbf{k}|^2} . \tag{3}$$

One then uses Eq. (2) to eliminate A_μ^{self} from the action, resulting in a nonlinear integrodifferential equation which can be solved by an iterative procedure using as zeroth-order trial functions the eigenfunctions of the Schrödinger equation in only the external field, A_μ^{external}. In the first iteration the Lamb shift appears as the real part of a complex energy shift given by [7]

$$E^{\text{self}} = -\frac{\alpha}{\pi}\frac{1}{m_0^2}\sum_{m,n}\int d^3k \frac{{}_nT_m{}^i(\mathbf{k})_m T_n{}^j(-\mathbf{k})}{\omega_{nm}^2 - |\mathbf{k}|^2} \times (\delta_{ij} - \hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j) , \tag{4}$$

where a sum is implied over $i,j=1,2,3$ and n,m stand for all quantum numbers of the nth or mth level, respectively, and run over all the eigenstates of the system (i.e., $n=nlm$ for hydrogen). In units where $\hbar=c=1$, $\omega_{nm} := E_n - E_m$ is an energy level difference (where $a:=b$ means a is defined by b), the $\hat{\mathbf{k}}_i$ are the components of a unit vector in the $\mathbf{k}$ direction, m_0 is the (reduced) electron mass, and the $\mathbf{T}$'s are electron form factors given by

$${}_n\mathbf{T}_m(\mathbf{k}) = \int dx^3 \phi_n^*(\mathbf{x})\frac{\nabla}{i}\phi_m(\mathbf{x})e^{i\mathbf{k}\cdot\mathbf{x}} , \tag{5}$$

where the ϕ_n form a complete set of electron wave functions for an atom, harmonic oscillator, cyclotron orbit, etc. If the dipole approximation (DA) is valid then $e^{i\mathbf{k}\cdot\mathbf{x}} \cong 1$ and, hence, ${}_n\mathbf{T}_m(\mathbf{k}) \cong \mathbf{p}_{nm}$, the matrix elements of the momentum, related to those of the position operator by $\mathbf{p}_{nm} = i\omega_{nm} m_0 \mathbf{r}_{nm}$.[29]

It is clear then that the introduction of a boundary will modify the Green's function used in Eq. (2), and thus the Lamb shift of an electron will depend on the Green's function of its environment. For simple boundary geometries it is well known that the electrostatic method of images generalizes to the full electromagnetic field as a technique for constructing Green's functions.[26,30,31] If $D_{\mu\nu}$ is the free-space Green's function, then the correct Green's function in the neighborhood of a perfectly conducting boundary, $\tilde{D}_{\mu\nu}$, will be some linear combination of $D_{\mu\nu}$ and additional image functions. Along with $\tilde{D}_{\mu\nu}$ we can then compute the modified electron form factors $\tilde{\mathbf{T}}$, and use them in Eq. (4) to evaluate the boundary-induced energy shift defined by $\Delta E := \tilde{E} - E_0$, where $\tilde{E}$ is the self-energy of the system near the conductor and E_0 the free-space value.

In general ΔE will be some function of the distance R, the boundary-system separation. The system then experiences a net van der Waals force given by $F = -\partial_R(\Delta E)$. In addition ΔE will contain both state-dependent and state-independent terms. The state-independent terms will contribute to the van der Waals force, but not to the Lamb shift since all levels will be shifted by the same amount (only level *differences* can be measured). Only state-dependent terms in ΔE, dependent on the quantum numbers n and m, will contribute to frequency shifts.

III. ATOM NEAR AN INFINITE PLANE

As a simple illustration of these ideas we now consider a neutral hydrogen atom a distance R away from a perfectly conducting infinite plane. We place the plane normal to the z axis at $z=0$, and the atom along the positive z axis at $z=R$. The plane may then be replaced with a charged-reversed image atom at $z=-R$, which amounts to a simple translation of the origin in the Green's function $D_{\mu\nu}(n)$ to $z=\pm R$ and a sign charge on the image function. If, in addition, the electron in the real atom has a momentum operator $\mathbf{p}\propto\nabla=(\partial_x,\partial_y,\partial_z)$, then the image momentum in our coordinates will be given by $\mathbf{p}'\propto\nabla':=(\partial_x,\partial_y,-\partial_z)$. We may incorporate both these changes into the form factor of (5) by making the replacement

$$
\begin{aligned}
{}_nT_m{}^i(\mathbf{k})\to{}_nT_m{}^i(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{R}}-[{}_nT_m{}^i(\mathbf{k})]'e^{i\mathbf{k}\cdot\mathbf{R}}\\
:={}_n\tilde{T}_m{}^i(\mathbf{k})\ ,
\end{aligned}
\tag{6}
$$

with $\mathbf{R}=(0,0,R)$ and $\mathbf{T}'$ a functional of ∇' rather than ∇. The form factor product needed in Eq. (4) becomes

$$
\begin{aligned}
{}_nT_m{}^i(\mathbf{k})_mT_n{}^j(-\mathbf{k})\to{}_n\tilde{T}_m{}^i(\mathbf{k})_m\tilde{T}_n{}^j(-\mathbf{k})\\
={}_nT_m{}^i(\mathbf{k})_nT_m{}^j(-\mathbf{k})\\
-\cos(2\mathbf{k}\cdot\mathbf{R})_nT_m{}^i(\mathbf{k})_mT_n{}^j(-\mathbf{k})\ .
\end{aligned}
\tag{7}
$$

In order to obtain Eq. (7) we have used symmetry in dummy summation and integration variables [a double integration is implicit from the definition of $\mathbf{T}$ in Eq. (5)] to combine two terms. Also factors of the form TT' have been multiplied by $\frac{1}{2}$ and those of $T'T'$ dropped altogether; these modifications are artifacts of the image procedure.[32] We shall call Eq. (4) with the above replacement Eq. (4′).

We may now proceed with the angular integration of (4′), and we assume the products TT and TT' to be functions of $k^2=|\mathbf{k}|^2$. (As noted elsewhere,[22] symmetry in the dummy variables can be used to justify this assumption.) This same symmetry allows us also to write the fraction

$$
\begin{aligned}
\frac{1}{\omega_{nm}^2-k^2}&=\frac{1}{2\omega_{nm}(\omega_{nm}-k)}+\frac{1}{2\omega_{nm}(\omega_{nm}+k)}\\
&\underset{\sum_{n,m}}{=}\frac{1}{\omega_{nm}(\omega_{nm}-k)}
\end{aligned}
\tag{8}
$$

in Eq. (4′) [where $\underset{\sum_{n,m}}{=}$ means with respect to the sum $\sum_{n,m}$]. The boundary-induced shift in the self-field energy, $\Delta E:=\tilde{E}-E_0$, is then

$$
\begin{aligned}
\Delta E^{\text{self}}=-\frac{\alpha}{\pi m_0^2}\sum_{n,m}\int dk\,{}_n\mathbf{T}_m(\mathbf{k})\cdot{}_m\mathbf{T}_n(-\mathbf{k})\\
\times\left[(1-\zeta_{nm})\frac{\sin(2kR)}{2kR}+(1+\zeta_{nm})\left[\frac{\cos(2kR)}{(2kR)^2}-\frac{\sin(2kR)}{(2kR)^3}\right]\right]\frac{k^2}{\omega_{nm}(k-\omega_{nm})}\ ,
\end{aligned}
\tag{9}
$$

where ζ_{nm} is a function of k^2 defined by $\zeta_{nm}(k^2):={}_nT_m^z(\mathbf{k})_mT_n^z(-\mathbf{k})/{}_n\mathbf{T}_m(\mathbf{k})\cdot{}_n\mathbf{T}_m(-\mathbf{k})$, which in the DA reduces to $\zeta_{nm}\cong|z_{nm}|^2/|\mathbf{r}_{nm}|^2$. ($\zeta_{nm}$ is introduced to display the asymmetry with respect to the z axis in a compact fashion.) We may extract from Eq. (9) the contribution to a single energy level by expanding $\Delta E=\sum_n\Delta E_n$, where $\Delta E_n=\sum_m\Delta E_{nm}$ in an obvious notation. At this stage we note that formula (9) is exact, as we have not yet employed the dipole approximation (DA).

To perform the remaining integration we will need to distinguish between the two cases $m\gtrless n$ since

$$
\omega_{nm}(k-\omega_{nm})=\begin{cases}-|\omega_{nm}|(k+|\omega_{nm}|), & m>n\\ |\omega_{nm}|(k-|\omega_{nm}|), & m<n\end{cases}
\tag{10}
$$

and in particular the case $m<n$ will be determined by a complex contour integration [see the Appendix, Eq. (A4)]. We also pass to the dipole approximation to simplify the form of the functions $\mathbf{T}(\mathbf{k})$. (These form factors can be calculated in practice, yielding an exact final answer, but the dipole approximation is good enough for our purpose here.) The dipole approximation, we recall is valid when the transition wavelengths λ_{nm} contributing to E_n are large compared to atomic dimensions; for hydrogenlike atoms when $Z\ll137$.[29] In the DA then, the nth level self-energy shift can be written as the sum of three terms $\Delta E_n^{\text{self}}=\Delta E_n^a+\Delta E_n^b+\Delta E_n^c$, where

$$
\Delta E_n^a=\frac{\alpha}{2}\sum_m|\omega_{nm}|^3|\mathbf{r}_{nm}|^2\frac{1+\zeta_{nm}}{\alpha_{nm}^3}\ ,
\tag{11}
$$

$$
\Delta E_n^b=\frac{\alpha}{\pi}\sum_m\omega_{nm}^3|\mathbf{r}_{nm}|^2\left[(1-\zeta_{nm})\left[\frac{1}{a_{nm}^2}-\frac{f(a_{nm})}{a_{nm}}\right]+(1+\zeta_{nm})\left[\frac{g(a_{nm})}{a_{nm}^2}+\frac{f(a_{nm})}{a_{nm}^3}\right]\right]\ ,
\tag{12}
$$

$$
\Delta E_n^c=\alpha\sum_{m(<n)}\omega_{nm}^3|\mathbf{r}_{nm}|^2\left[(1-\zeta_{nm})\frac{\cos a_{nm}}{a_{nm}}-(1+\zeta_{nm})\left[\frac{\sin a_{nm}}{a_{nm}^2}+\frac{\cos a_{nm}}{a_{nm}^3}\right]\right]\ ,
\tag{13}
$$

where we have defined $a_{nm}:=2R|\omega_{nm}|$ and the functions f and g are defined as per Abramowitz and Stegun;[33] they are related to the Fresnel sine and cosine integrals [see the Appendix, Eq. (A1)].

In addition to the energy shift ΔE^{self} due to the change of the self-field, there is also the usual electrostatic London energy arising from the instantaneous dipole moment of the atom interacting with its image in the wall. This term is obtained if we consider the atom and its image as interacting through an electrostatic potential $V(\mathbf{r},\mathbf{r}',\mathbf{R},-\mathbf{R})$ between all four charges (see for example Schiff[34]). Expanding V for large R and application of second-order perturbation theory gives the London energy as

$$E_n^{\text{London}}=-\frac{\alpha}{16R^3}\sum_m(|x_{nm}|^2+|y_{nm}|^2+2|z_{nm}|^2)=-\frac{\alpha}{2}\sum_m|\omega_{nm}|^3|\mathbf{r}_{nm}|^2\frac{1+\zeta_{nm}}{a_{nm}^3}\,, \tag{14}$$

with the convention $|\omega_{nm}|/|\omega_{nm}|=1$. Note that $E_n^{\text{London}}=-\Delta E_n^a$, and so these two cancel out exactly in the total energy given by $\Delta E_n^{\text{total}}=\Delta E_n^{\text{self}}+E_n^{\text{London}}=\Delta E_n^b+\Delta E_n^c$ or

$$\Delta E_n^{\text{total}}=\frac{\alpha}{\pi}\sum_m\omega_{nm}^3|\mathbf{r}_{nm}|^2\left\{(1-\zeta_{nm})\left[\frac{1}{a_{nm}^2}-\frac{f(a_{nm})}{a_{nm}}+\pi\Theta_{nm}\frac{\cos a_{nm}}{a_{nm}}\right]\right.$$
$$\left.-(1+\zeta_{nm})\left[\frac{g(a_{nm})}{a_{nm}^2}+\frac{f(a_{nm})}{a_{nm}^3}-\pi\Theta_{nm}\left[\frac{\sin a_{nm}}{a_{nm}^2}+\frac{\cos a_{nm}}{a_{nm}^3}\right]\right]\right\}, \tag{15}$$

where we define $\Theta_{nm}:=\Theta(n-m)$, the customary unit step function, which "switches on" only if n is an excited state. Equation (15) is our primary result; it is the energy-level shift additional to the free-space Lamb shift that occurs in the vicinity of the plate.

IV. LIMITING CASE $R\to 0$

Formula (15) within the DA is exact for all atom-plate separations R. To compare with the literature, it is expedient to consider the limits $R\to 0$ ($a_{nm}\ll 1$) and $R\to\infty$ ($a_{nm}\gg 1$); the former is taken up first.

We will make use of the asymptotic expansions of f and g as given in the Appendix, Eq. (A6). For $R\to 0$ Eq. (15) becomes

$$\Delta E_n^{\text{total}}(R\to 0)\sim-\frac{\alpha}{\pi}\sum_m\omega_{nm}^3|\mathbf{r}_{nm}|^2\left[\frac{2\zeta_{nm}}{a_{nm}^2}+(1-3\zeta_{nm})\frac{\pi}{4a_{nm}}-(1+\zeta_{nm})\frac{\pi}{2a_{nm}^3}\right.$$
$$\left.-\tfrac{2}{3}(1-2\zeta_{nm})\ln(a_{nm})+\pi\Theta_{nm}\left[\frac{2\zeta_{nm}}{a_{nm}}+\frac{1+\zeta_{nm}}{a_{nm}^3}\right]+O(1)\right], \tag{16}$$

in which we have neglected terms of orders 1, R, R^2, and so on. To bring this into a more familiar form and to separate state-independent from state-dependent terms we now employ the Thomas-Reiche-Kuhn (TRK) sum rule and the closure relation given by[29] ($|\mathbf{p}_{nm}|^2=m_0^2\omega_{nm}^2|\mathbf{r}_{nm}|^2$)

$$\begin{aligned}&\sum_m\omega_{nm}|\mathbf{r}_{nm}|^2\equiv-\frac{1}{2m_0}\quad\text{(TRK)}\,,\\&\sum_m|\mathbf{r}_{nm}|^2\equiv\langle n|r^2|n\rangle\quad\text{(closure)}\,,\\&\sum_m|\mathbf{p}_{nm}|^2\equiv\langle n|p^2|n\rangle\quad\text{(closure)}\,.\end{aligned} \tag{17}$$

Equation (16) can then be written

$$\begin{aligned}\Delta E_n^{\text{total}}(R\to 0)=&-\frac{\alpha}{16R^3}\langle n|r^2+z^2|n\rangle+\frac{\alpha}{4\pi m_0R^2}\\&+\frac{\alpha}{8R}\langle n|p^2-3p_z^2|n\rangle-\frac{2\alpha}{3\pi}\sum_m(|\mathbf{r}_{nm}|^2-2|z_{nm}|^2)\ln(2R|\omega_{nm}|)\\&-\frac{\alpha}{R}\sum_{m(<n)}|\omega_{nm}|^2|z_{nm}|^2-\frac{\alpha}{8R^3}\sum_{m(<n)}(|\mathbf{r}_{nm}|^2+|z_{nm}|^2)\\=&:E_n^{\text{London}}+E_n^{(1)}+E_n^{(2)}+E_n^{(3)}+E_n^{(4)}+E_n^{(5)}\,.\end{aligned} \tag{18}$$

We see that the leading term is once again the London energy E_n^{London} which becomes altered slightly for excited states $(n>0)$ by the last term $E_n^{(5)}$. (In particular, contributions to the sum $\sum_m$ in E_n^{London} are enhanced by a factor of 2 if $m<n$.) For $n=0$ this formula agrees precisely with the original results of Casimir and Polder[13] as well as a later computation done by Barton.[35] Casimir and Polder did not consider excited states, and our result here differs only slightly from that of Barton's for this case. (In particular, he does not seem to keep the terms proportional to Θ_{nm} in this limit, $R\to 0$. He does give a good discussion of the physical interpretation of all these terms, and the reader is referred thither.) Notice that $E_n^{(2)}$ is independent of the quantum number n and so it will not contribute to the Lamb shift. It will, as noted before, contribute to the van der Waals force via $F=\partial_R(\Delta E)$.

In this $R\to 0$ limit the shift between the 2s and 1s levels of hydrogen, for example, can be computed exactly, the result being

$$\Delta E_{200}-\Delta E_{100}=-\frac{13}{4}\frac{e^2}{a_B}\left[\frac{a_B}{R}\right]^3 \quad (R\to 0)\ , \tag{19}$$

where a_B is the Bohr radius. For macroscopic distances R this shift is of course unobservably small. However, if R were a few Bohr radii, the effect might be measured. We note that in the scanning tunneling electron microscope, single atoms at the tip of an ultrafine needle are suspended a few atomic radii above a planar (metallic) crystal surface.[23]

V. LIMITING CASE $R\to\infty$

The limit $R\to\infty$ is the more interesting case, as here the radiation retardation comes into play. In fact Casimir and Polder were primarily concerned with this limit. It will also be one proof of the self-energy formulation of QED if we can recover the correct results in the radiation far field (these results are usually ascribed to vacuum fluctuations of the second-quantized radiation field).[36]

The needed asymptotic expansions are again given in the Appendix [see Eqs. (A5)]. For the total energy shift we then get from Eq. (15)

$$\Delta E_n^{\text{total}}(R\to\infty)\sim\frac{\alpha}{\pi}\sum_{m(\neq n)}\omega_{nm}^3\,|\mathbf{r}_{nm}|^2\left\{\frac{4}{a_{nm}^4}+\pi\Theta_{nm}\left[(1-\zeta_{nm})\frac{\cos a_{nm}}{a_{nm}}-(1+\zeta_{nm})\left[\frac{\sin a_{nm}}{a_{nm}^2}+\frac{\cos a_{nm}}{a_{nm}^3}\right]\right]\right\}+O\left[\frac{1}{R^5}\right]. \tag{20}$$

The first term in this sum is of order R^{-4} and is precisely the Casimir Polder energy, first obtained in 1948. Notice that there are no longer any ground-state contributions of order R^{-3}—the London energy has been completely canceled by retardation effects. The additional terms, proportional to sine and cosine of $2R\,|\omega_{nm}|$, switch on only for excited states. They agree with similar corrections obtained by Barton.[35] It is usual to write the Casimir-Polder term as

$$\Delta E_n^{\text{CP}}=-\frac{\hbar c}{8\pi R^4}\gamma_n\ , \tag{21}$$

where γ_n is the atomic polarization of level n defined by

$$\gamma_n:=-e^2\sum_{m(\neq n)}\frac{2\,|\mathbf{r}_{nm}|^2}{\omega_{nm}}\ , \tag{22}$$

where $\alpha=e^2/\hbar c$. Notice that ΔE_n^{CP} depends only on the constants $\hbar$, c, and the polarization γ. Since e^2, the electromagnetic coupling constant, does not enter—usually this energy is viewed as being due solely to the vacuum zero-point fluctuations.[1] Yet it was derived here in the absence of any such fluctuations. For the difference between the 1s and 2s levels of hydrogen, formula (20) results in

$$|\Delta E_{100}-\Delta E_{200}|\simeq\frac{e^2}{a_B}\left[\frac{a_B}{R}\right]^4 \quad (R\to\infty) \tag{23}$$

up to some leading unitless constant on the order of unity. Again we see the power law change, from R^{-3} to R^{-4}, a retardation effect. This change in the van der Waals power law at large distance was first observed experimentally, which was the original motivation for the theoretical investigation of Casimir and Polder.[13]

VI. CONCLUSIONS

We have seen how effects normally attributed to vacuum fluctuations in the second-quantized, linear theory of the radiation field can be equally well computed within the framework of a nonsecond quantized but nonlinear theory which is based on self-fields. The information "lost" by not second quantizing could very well be "found" in the nonlinearities of the new formulation. Like the Schrödinger and Heisenberg pictures of quantum mechanics, perhaps these are simply two versions of the same theory.

Our program is to see how far we can go in understanding radiative processes without second quantization or vacuums which fluctuate. Work has been completed on spontaneous emission near boundaries,[22] and is in progress on the general Casimir-Polder force between two atoms, the Casimir parallel-plate effects, the Unruh effect of accelerating vacua, and boundary effects on the measured value of $g-2$ in Penning traps.

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APPENDIX

Abramowitz and Stegun[33] define the functions f and g as

$$\begin{aligned} f(z) &:= \int_0^\infty \frac{\sin t}{t+z}\,dt \ , \\ g(z) &:= \int_0^\infty \frac{\cos t}{t+z}\,dt \ . \end{aligned} \tag{A1}$$

If one defines the complementary Fresnel sine and cosine integrals as

$$\begin{aligned} \mathrm{Si}(z) &:= \int_z^\infty \frac{\sin t}{t}\,dt \quad (\,|\arg z\,| < \pi) \ , \\ \mathrm{Ci}(z) &:= \int_z^\infty \frac{\cos t}{t}\,dt \quad (\,|\arg z\,| < \pi) \ , \end{aligned} \tag{A2}$$

then one can obtain by analytic continuation the identities

$$\begin{aligned} f(z) &\equiv \mathrm{Si} z \cos z - \mathrm{Ci} z \sin z \quad (\,|\arg z\,| < \pi) \ , \\ g(z) &\equiv \mathrm{Ci} z \cos z + \mathrm{Si} z \sin z \quad (\,|\arg z\,| < \pi) \ . \end{aligned} \tag{A3}$$

The following integrals are also needed, and may be done by contour integration (the result is tabulated in Gradshteyn and Ryzhik[37])

$$\begin{aligned} \int_0^\infty \frac{\sin t}{t-z}\,dt &= -f(z) + \pi \cos z \quad (\mathrm{Re} z > 0) \ , \\ \int_0^\infty \frac{\cos t}{t-z}\,dt &= g(z) - \pi \sin z \quad (\mathrm{Re} z > 0) \ . \end{aligned} \tag{A4}$$

Finally, we will also need the following asymptotic expansions, which can be obtained from Abramowitz and Stegun also [in the case of Eqs. (6), after some work]

$$\begin{aligned} f(z) &\sim \frac{1}{z}\left[1 - \frac{2!}{z^2} + \frac{4!}{z^3} - \cdots\right] \quad (\,|\arg z\,| < \pi) \ , \\ g(z) &\sim \frac{1}{z^2}\left[1 - \frac{3!}{z^2} + \frac{5!}{z^4} - \cdots\right] \quad (\,|\arg z\,| < \pi) \ , \end{aligned} \tag{A5}$$

where $|z| \to \infty$ and

$$\begin{aligned} f(z) &\sim \frac{\pi}{2} - \frac{\pi}{4}z^2 - z + \tfrac{29}{36}z^2 \\ &\quad + (\gamma + \ln z)\left[\frac{z^3}{c} - z\right] + O(\,|z|^4) \ , \\ g(z) &\sim (\gamma + \ln z) - \tfrac{1}{2}z^2 \ln z + \frac{\pi}{2}z + O(\,|z|^2) \ , \end{aligned} \tag{A6}$$

where $|z| \to 0$ and where γ is Euler's number and the principle branch of $\ln(z)$ is used.

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