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Quantum electrodynamics based on self-energy versus quantization of fields: Illustration by a simple model

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The relation between the formulations of quantum electrodynamics based on self-energy and the standard perturbative quantum electrodynamics is further elaborated and a simpler model is given.

In the preceding Comment Bialynicki-Birula[1] argues that our formulation of quantum electrodynamics based on the self-energy,[2] although not explicitly using the second-quantization operator formalism for both the electromagnetic field A_μ and the matter field ψ, does nevertheless implicitly contain some elements of field quantization.

I think it is useful and important to discuss the relations between the standard QED perturbation theory and our formulation, which we think to be an independent self-consistent formulation on its own right. There must indeed be some close relationships since they both refer to the same physical phenomenon and to the same results. Consequently, I shall answer the specific points raised in the preceding paper and try to clarify the distinct features of both formulations.

The first question concerns the meaning of the Fourier expansion of the matter field, which we certainly can perform, in the form

$$\psi(\mathbf{x},t)=\sum_n \psi_n(\mathbf{x})e^{-iE_n t} . \quad (1)$$

To answer it I shall consider a very simple model which is instructive and has exactly all the elements of our method and leads to the final results in a few lines. The model describes a complex scalar field ψ with a Hamiltonian H_0 and an interaction with another real scalar field ϕ with the action

$$W=\int d\mathbf{x}\int dt\left[\psi^*\left[i\frac{\partial}{\partial t}-H_0\right]\psi-\lambda\psi^*\psi\phi+\tfrac{1}{2}\phi^2\right] . \quad (2)$$

Eliminating ϕ (as we eliminated A_μ in the electrodynamic case) by using the equation of motion for ϕ: $\phi=\lambda\psi^*\psi$, we obtain the nonlinear action

$$W=\int d\mathbf{x}\int dt\left[\psi^*\left[i\frac{\partial}{\partial t}-H_0\right]\psi-\frac{\lambda^2}{2}\psi^*\psi\psi^*\psi\right] . \quad (3)$$

For simplicity this model has no Green's function for the ϕ field. The Fourier expansion (1) inserted into (3), and with the t integration performed, gives

$$W=\int d\mathbf{x}\sum_{n,m}\psi_n^*(E_m-H_0)\psi_m\delta(E_n-E_m)$$
$$-\frac{\lambda^2}{2}\sum_{n,m,r,s}\delta(E_n-E_m+E_r-E_s)\psi_n^*\psi_m\psi_r^*\psi_s . \quad (4)$$

We see that the action is a *sum* of contributions of all Fourier coefficients. In the limit of zero interaction ψ_n's are the solutions of H_0 (action W containing $[i(\partial/\partial t)-H]\,\psi_n$ vanishes when solutions of the equations of motion are inserted). Thus the theory gives us the physical interpretation of Fourier coefficients $\psi_n(\mathbf{x})$ in Eq. (1). Bialynicki-Birula thinks that we should have written instead of (1)

$$\psi(\mathbf{x},t)=\sum_n c_n\psi_n(\mathbf{x})e^{-iE_n t} ;$$

not all $c_n=1$. Up to this point this does not make any difference, because one can always replace $c_n\psi_n(\mathbf{x})$ by $\psi_n'(\mathbf{x})$ everywhere conceptually. One should keep apart the Fourier expansion of a field from the coherent single-particle state, or superposition of probability amplitudes. We use $e\psi^*\psi$ as the real charge density of the particle which produces its own self field (as Schrödinger wanted to use it) and not simply as a probability density. This is connected with the fundamental problem of the interpretation of the ψ field in quantum theory.

With the interaction present we solve Eq. (4) now *by iteration*. We choose to lowest order of iteration $\psi_n=\psi_{n\alpha}^0$, the eigenstates of H_0 where α is a degeneracy index which was suppressed in Ref. 2 which we now restore to answer the technical objection raised at the end of Ref. 1. We then write $E_{n\alpha}=E_{n\alpha}^0+\Delta E_{n\alpha}$ and assume in lowest order of iteration that $\Delta E_{n\alpha}$ is a number. Then in the first term of Eq. (4), because of the orthogonality of $\psi_{n\alpha}^0$, we just get $\Delta E_{n\alpha}$, canceling an infinite factor $\delta(E_n-E_m)$ from both sides of (4). But in the second term of (4) in sums over m,r,s we must also sum over all degeneracy indices $\beta\gamma\delta$ of the unperturbed wave functions. The two ways of satisfying the δ-function constraint in the second term of (4), which in the case of electrodynamics were identified with the vacuum-polarization and Bethe terms,[2] give here in this simple case the same result and we obtain the final expression

$$\Delta E_{n\alpha}=\lambda^2\int d\mathbf{x}\left[\psi_{n\alpha}^*\psi_{n\beta}\sum_{\substack{m\\ \gamma,\delta}}\psi_{m\gamma}^*\psi_{m\delta}\right] . \quad (5)$$

I agree that the sum over a complete set of states looks like a field quantization, but we do such sums in first-quantized quantum theory all the time. The whole formalism of second-quantized field operators has not been used at all. The sum arises because the field equations are nonlinear in ψ.

This leads us to the next question in Ref. 1—whether this theory will be the same as perturbation theory in higher-than-one-loop corrections. This is difficult to answer since we have an iteration procedure. In the next order of iteration we have to take the effective potential resulting from (5), i.e.,

$$\Delta V(\mathbf{x})=\lambda^2 \sum_{\substack{m\\ \gamma,\delta}} \psi^*_{m\gamma}(\mathbf{x})\psi_{m\delta}(\mathbf{x}) , \tag{6}$$

and solve the new equation

$$[E-H_0-\Delta V(\mathbf{x})]\psi^{(1)}=0 \tag{7}$$

and evaluate $\Delta E^{(1)}$ with these new wave functions $\psi^{(1)}$. We have no results so far on the comparison of higher order of iterations with perturbation theory.

Coming back to electrodynamics, we employ a first-quantized Dirac field ψ. There is a consistent interpretation of antiparticles in the first quantized theory and the use of Stückelberg-Feynman Causal Green's function $D_C(x-y)$, which we adopt as correctly observed in Ref. 1, is related to the Dirac ψ as the source of the self-field. It is true that the self-field produced by the Dirac current and with the use of D_C is complex but as we show, the imaginary part of the self-field accounts precisely for the spontaneous emission. Only in classical electrodynamics with point particles is the self-field real and one can take retarded Green's functions but then there is no spin and no antiparticles in the source. Bialynicki-Birula quotes a paper by Feynmann according to which "the elimination of A_μ with the use of the Feynman propagator is completely equivalent to quantizing A_μ." This agrees with our point of view that all electromagnetic field originates from the self-field of electrons and that the quantized properties of A_μ reflects the first quantization of the sources.[3] But in contrast, the current practice assumes an independent *existence* for a quantized omnipotent radiation field, with its vacuum fluctuations, whereas our formulation puts the quantum fluctuations onto the electron.

Of course I am aware of the many discussions on the relations between radiation reaction and vacuum fluctuations. And there must be such relations. And with such relations in mind, we wanted to carry out explicitly the formulation of electrodynamics based solely on self-energy, without the machinery of second quantized ψ- and A_μ-field operators and its inevitable perturbative approach. Whether some elements of this theory look like the second quantization is secondary. The important thing is to find out which formulation will eventually lead us to a complete finite quantum electrodynamics. We believe we have a very simple and direct formulation which unifies all radiative processes (Lamb shift, spontaneous emission, anomalous magnetic moment, etc.) in one formula, is free of the infrared problem and allows the calculation of these effects to all orders in $(Z\alpha)$ in closed form and incorporates a very simple regularization procedure.[4]

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[1] I. Bialynicki-Birula, preceding comments, Phys. Rev. A **34**, 3500 (1986).

[2] A. O. Barut and J. F. Van Huele, Phys. Rev. A 32, 3187 (1985); A. O. Barut and J. Kraus, Found. Physics 13, 189 (1983).

[3] Compare also A. O. Barut, in *Quantum Theory and the Structure of Space-Time,* edited by L. Castell (Hauser, Munich, 1986).

[4] A. O. Barut and J. Kraus (unpublished).