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STABLE PARTICLES AS BUILDING BLOCKS OF MATTER

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Only absolutely stable indestructible particles can be truly elementary. A simple theory of matter based on the three constituents, proton, electron and neutrino (and their antiparticles), bound together by the ordinary magnetic forces is presented, which allows us to give an intuitive picture of all processes of high-energy physics, including strong and weak interactions, and make quantitative predictions.

I. INTRODUCTION

At present, the picture of elementary particle physics mostly used in high-energy phenomenology is becoming admittedly very complicated. Besides leptons (which we see), one introduces families of "quarks", each with different colours, then the so-called "gluons", which are the gauge vector mesons binding the quarks, then there are the so-called "Higgs particles", which give masses to some of the vector mesons (all of which are not seen in the laboratory). One is already beginning to talk about a second generation of more fundamental and simpler objects for these quarks and gluons etc., even though these first generations of "basic" objects have not been seen. This type of framework seems to create more problems than it solves [1].

Against this background of recent developments, we wish to expand here a very intuitive and simple physical theory, along the traditions of atomic and nuclear structure theories, from which a unified picture of high-energy phenomena can be deduced. High-energy physics is very expensive. One must have alternative views, if only to test better the inevitability of the orthodox picture [2]. Furthermore, physical phenomena must be explainable in a simple intuitive form in terms of already verified definite primary concepts, and continuous with the existing physics.

II. THE PHYSICAL PRINCIPLES

Atoms and molecules are best described as built from electrons and nuclei bound by Coulomb forces because they disintegrate into electrons and nuclei, which we detect, and because these constituents are stable as far as atomic processes are concerned. In turn, nuclei and all the hadrons eventually decay into the absolutely stable particles: protons, electrons, neutrinos and photons (electromagnetic field). From the point of view of S-matrix theory, the asymptotic states are at the end the stable particles, and <u>all</u> other unstable particles must occur as resonance poles in the S-matrix between these stable particles. This is a theorem. Is it not simpler, therefore, to describe all the intermediate phenomena as a dynamical problem between the stable particles, instead of introducing new hypothetical particles? (Or, how must the interactions between the stable particles look like to account for the observed phenomena?) If p, e, ν, are indestructible, they must be also indestructible inside the hadrons. We present here a theory in which all matter is made up of these stable constituents, bound again by electromagnetic forces.

One can of course ask questions about the nature of the absolutely stable particles themselves. This is another level of enquiry. In this paper we shall take these as given and elementary.

At first such an idea might seem impossible or outrageous, because electromagnetic forces between p, e and ν (and their antiparticles) cannot possibly, one would think, give the necessary strong binding and strong interactions between hadrons, and so-called weak forces. On the other hand, the idea that stable particles are the constituents of hadrons is probably very old as a general idea, if not carried out in specific details. For example, with the hypothesis of neutrino in β decay, Pauli's model of the neutron was a bound state proton, electron and antineutrino [3]. This model was soon abandoned (to be revived much later [4]) for one did not know how to suppress the large magnetic moment of the electron (on nuclear scale) inside the nucleus, and one did not know any deep enough well to contain or confine the electron inside the nucleus.

What is new, however, is the recognition that magnetic forces between the stable particles, when treated non-perturbatively, become very strong at short distances (short-ranged), provide a deep enough well to give rise to high mass narrow resonances, have saturation property and lead, by magnetic pairing, to the compensation of the large magnetic moment of the electron. In the construction of atoms and molecules we make use only of the electric (Coulomb) part of the electromagnetic forces and treat magnetic forces as small perturbations. There is, however, another regime of energies and distances in which magnetic forces play the dominant role and the electric forces are small perturbations. We shall show this duality with explicit calculations. It would have been strange if Nature provided magnetic forces just to be tiny corrections to the building principle of atoms and molecules (which could exist without them) and not to play an equally important role in the structure of matter. Clearly, a model of this type also automatically provides a dynamical theory of nuclear forces.

Even from the point of view of perturbative QED one knows that QED is, asymptotically a strong coupling theory: the effective coupling constant increases with momentum transfer. Hence, one may end up with structures giving rise to new states. The whole of electromagnetic interactions cannot be simply disposed of by calculating a lowest order one-photon-exchange diagram.

There are two main immediate questions or objections to our propositions. (i) Why do we not see in the laboratory strong forces between proton and electron, electron and positron, or electron and neutrino etc., whereas we see strong forces between pions and protons, or protons and neutrons etc.? (ii) How can we obtain the rich world of hadrons just starting from the three stable particles p, e, ν (and their antiparticles), the multitude of internal quantum numbers like isospin, strangeness, charm etc., the multiplet structures and symmetries?

Correspondingly, this work has two parts. A kinematical part showing the composition of all hadrons and their multiplet structures, hence the meaning of internal quantum numbers in terms of the stable particles, p, e, ν. This by itself is a remarkable mapping of hadron states onto the combinations of stable particles, the eventual final products of all unstable matter, and of hadron quantum numbers into those of three stable particles, p, e, ν. The second part is dynamical showing that ordinary magnetic spin-spin and spin-orbit forces, when treated non-perturbatively, have the correct strength and shape to give hadronic and nuclear states.

We begin with the second part in order to answer immediately the questions raised above.

The Dynamics of Magnetic Interactions

A number of models, with increasing complexity, have been studied in recent years, and we have a good understanding of the spin-spin and spin-orbit potentials at short distances [4) - 7)]. In Appendix I we discuss the results of these models and in Appendix II we give the analytical proof of the existence of resonances. Here we shall explain the main idea in terms of a simple case. Consider, for example, a relativistic charged spinless particle m in the field of a fixed (quantum) magnetic momentum μ [8)], or alternatively, a charged spin 1/2 particle of mass m and magnetic moment $\vec{\mu}$, in the field of a fixed charge [9)]. In both cases, the effective radial equation can be written, in appropriate coordinates, as

$$\left[-\frac{d^2}{dy^2} + V(j,\ell,r)\right]u = \lambda^2 u \qquad (1)$$

where the effective potential is given, apart from the Coulomb potential α/r, by

$$V(j,\ell,r) = \frac{\ell(\ell+1)}{y^2} + \epsilon\frac{2c(j,\ell)}{y^3} + \frac{1}{y^4}\ , \qquad (2)$$

with $\epsilon = \pm 1$ (relative sign of the charge and magnetic moment); $c(j,\ell)$ is equal to $-(\ell+1)$ for $\ell = j + 1/2$ and equal to ℓ for $\ell = j - 1/2$. Furthermore (in units $c = \hbar = 1$), $r = \mu e y = \mu_0(\alpha/2M)y$ (M is the mass of fixed magnetic moment - in the second case put $M = m$), and the eigenvalue λ is

$$\lambda^2=(E^2-m^2)\mu^2\alpha^2=(E^2-m^2)\mu_o^2\cdot\frac{\alpha^2}{4M^2} \qquad (3)$$

If we solve the same problem with a Dirac equation and give also an anomalous magnetic moment a to the particle, then additional terms are added to Eq. (2). [5] Further models also treat the magnetic moments of both of the particles.(App. I)

The potential (2) is treated in atomic phenomena (lately also in the quark model) as a perturbation. This is justified if the energies are of the order of Coulomb energies and for Coulombic bound state wave functions. New phenomena occur, however, if the magnetic potential is treated non-perturbatively. Fig. 1 shows the schematic form of the potential at two different energies and angular momenta and in the case when the anomalous magnetic moment terms are also included.

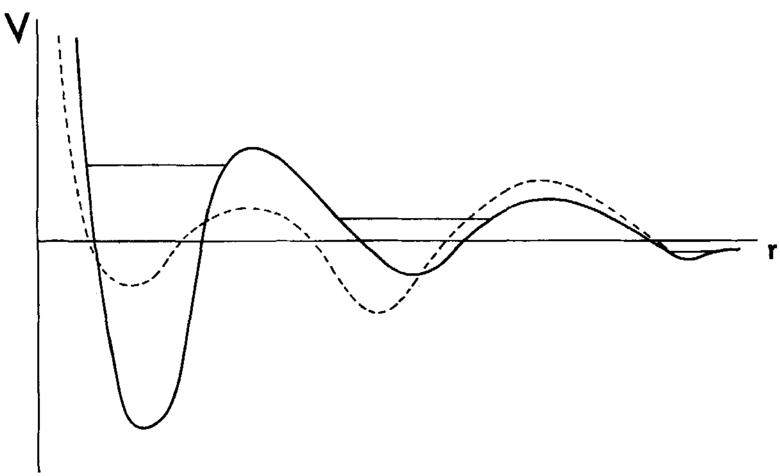


FIGURE 1. Schematic form of the effective radial magnetic potential V as a fuction of the radial distance r for two different fixed values of energy and angular momentum.

We see three distinct regions of potential wells: The Coulomb region at distances $r = 1/\alpha m$ (Bohr radius), hence momenta of the order of αm or non-relativistic energies of the order of $\alpha^2 m$, the nuclear region at $r \sim \alpha/m$, (relativistic) energies m/α (~ 70 MeV) and the supernuclear region of $r \sim \alpha^2/m$ and energies m/α^2 (10 GeV). The occurence of the energy scales $m/\alpha = 70$ MeV (and $m/\alpha^2 = 9.6$ GeV) is a characteristic of magnetic interactions. It is empirically known that masses of hadrons are integer or half integer multiples of m/α.

For unstable, hence positive energy, magnetic resonances the total mass is in general <u>greater</u> than the sum of the constituents, i.e. positive binding energy. This is an important difference from the usual intuition of a negative binding energy-composite systems.

The form of the potential at very short distances is still quite uncertain in these models. Furthermore, the potentials are modified by form factors. Form factors must also be calculated non-perturbatively, and self-consistently from the wave functions which are localized around each well, respectively, in Fig. 1, [6], [7] Form factors can easily be incorporated into the model (1) - (2) by taking $\mu = \mu(r)$. At intermediate distances the form of the potential is essentially correct. Unfortunately, quantum electrodynamics cannot tell us anything about the non-perturbative short distance behaviour of the potential between two particles.

Zero-mass Limit

It is important for our model later to remark that Eqs. (1) and (2) also hold for a massless particle in the field of a magnetic moment, or for a massless particle with an anomalous magnetic moment (or with only an anomalous form factor) in the field of a charge [10]. Note that mass m appears only in Eq. (3), setting the scale of the eigenvalue λ of Eq. (1). (Appendix III)

We can now answer the question as to why we apparently do not see strong interactions in the laboratory between the stable particles p, e, ν.

Scattering against a Barrier

The effect of large repulsive potential barriers as in Fig. 1 on the scattering of two fermions (say e^+, e^-)

can be evaluated numerically (and sometimes analytically). (Appendix IV) The cross-section of penetration to the attractive region is very small except at the sharp energy and angular momentum of the resonance, when "resonance penetration" [11] takes place. The partial phase shift, shown in Fig. 2, shows a sharp jump of about π near the resonance energy (anomalous scattering).

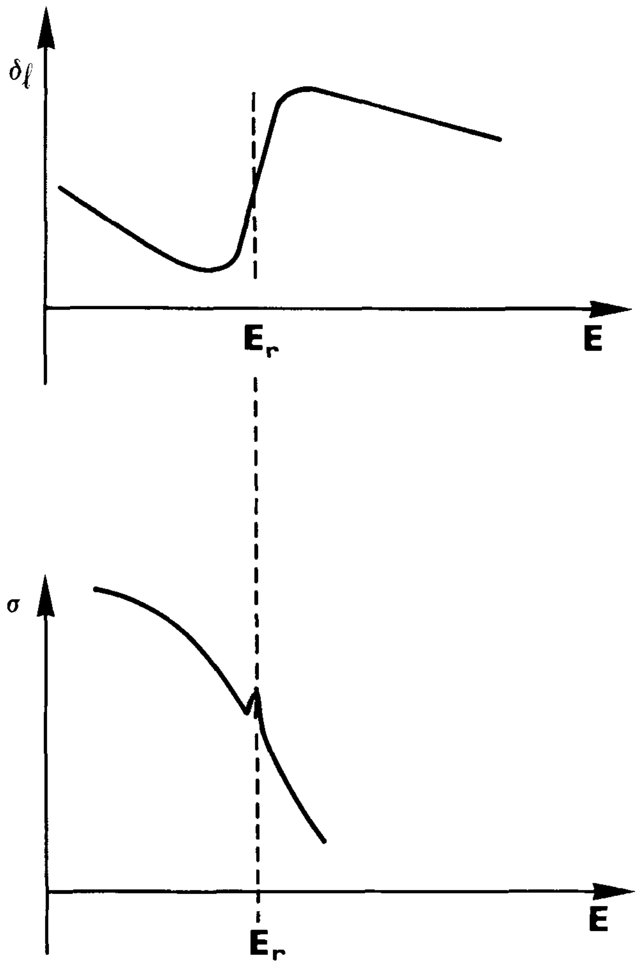


FIGURE 2. The effect of a repulsive barrier on the cross section σ around the resonance energy E_r.

The sharper the resonance, the steeper is the jump of the phase shift. The effect of this behaviour on the total cross section is, however, only a small bump, its width being proportional to the width of the resonance (Fig. 2). Indeed most hadron resonances are experimentally seen as such small bumps in cross sections on a large background. Some predictions based on this phenomenon will be made after we present the model of hadrons.

On the other hand, a pion, being itself a spin-zero resonance state of stable particles (see following sections), can penetrate much more easily into the region of strong magnetic forces of other hadron constituents, because of the absence of the spin-orbit barrier.

An important property of magnetic potentials (Fig. 1) is that the scattering amplitude is analytic in the whole of the angular momentum plane, hence is a sum of Regge pole contributions only. This has many applications in the analysis of scattering processes. (Appendix V)

III. ORDINARY AND STRANGE MATTER

Ordinary matter can be built up from p, e and ν (and their antiparticles) according to the rules that we shall state explicitly. These are pions, neutron and Δ^- resonances, hence also nuclear matter, atoms and molecules. In order to describe the building-up principle in a more general way to include "strange" particles, we must first talk about the μ meson. The μ meson can be thought of as a magnetic excitation of the electron due to the interaction of its anomalous magnetic moment with its own field. These arguments are at present semiclassical [12), 13)]. Another (perhaps equivalent) way, from our point of view, is to consider μ as a magnetic resonance state of $(e\nu\nu)$ into which it decays. We shall see that the pairs of the type $(e\bar{\nu})$ are identified with pions. Thus, in order to obtain a spin-1/2 state we need three stable particles, and $(e\nu\bar{\nu})$ should be then dynamically a little more stable than the $(e\bar{\nu})$ states.

The magnetic three-body problem $(e\nu\bar{\nu})$ can be approximated by an equivalent two-body problem $(e\bar{\nu})\nu$ and considerations similar to Eqs. (1) - (3) may be applied. The charge-magnetic moment system gives in the Bohr-Sommerfeld quantization a quantized energy spectrum of the form $\Delta E = \lambda n^4$, $n = 1, 2, 3, \ldots$ Adding this to the rest mass, one obtains a leptonic mass spectrum

$$M_N = m_e + \frac{3}{2} \frac{m_e}{\alpha} \sum_{n=0}^{N} n^4 \qquad (4)$$

for electron ($N = 0$), muon ($N = 1$), τ ($N = 2$),... The predictions for muon (105.55 MeV) and τ(1786.08 MeV) work very well and the next lepton predicted is δ (10.293 GeV). The relativistic version of Eq. (4) gives $n^4/(n^4+a)^{\frac{1}{2}}$. **The coefficient** 3/2 can also be derived by semiclassical arguments [12]. These results should only be considered as a beginning of a dynamical theory of heavy leptons. Nevertheless, they are interesting, because we have no other hints or ideas concerning the repetitions of leptons in the series $e, \mu, \tau, \ldots$, which is one of the most fundamental open problems of particle physics [14].

The ν-resonances are inferred as mentioned above from the $m = 0$ limit of the Dirac equation in models similar to Eqs. (1) - (3). Hence an interacting ν is necessarily a four-component neutrino. Only in the asymptotic region can the free Dirac equation be split into two two-component equations. We shall make the hypothesis that the neutrino has an anomalous magnetic moment, or at least a magnetic form factor, even if its magnetic moment is zero (on the mass shell). We also do not make, at this stage, a difference between ν_e and ν_μ (see Section VI)

The μ meson, behaving very much like the electron, can in turn form magnetic pairings and resonances with the stable particles, forming the so-called "strange" hadrons. In fact, it will turn out that the number of $\mu^\pm$ mesons in hadrons is exactly equal to the "strangeness" quantum number of hadrons. These apparently new types of hadrons are more unstable and decay into ordinary hadrons if the μ inside the hadron decays. During strong interactions, μ is stable, hence strangeness is conserved (see also next section). The μ meson, rather than being a "redundant" particle ("the world would be the same if μ did not exist"(!)) now plays an essential role in building up the hadrons. This process is then continued with the τ-excitations, etc.

IV. CONSTRUCTION OF HADRON STATES AND BUILT-IN CONSERVATION LAWS

There is a very simple relationship between lepton quantum numbers and quark quantum numbers. If we compare the triplet $\ell = (\nu, e^-, \mu^-)$ with the quark triplet $q = (u, d, s)$, we have

$$Q_q = Q_\ell + \frac{2}{3} B_\ell, \quad B_q = B_\ell - \frac{2}{3} B_\ell, \tag{5}$$

where B_ℓ stands for the lepton number and B_q for the baryon number. This we have called the "shifting principle": shifting two-thirds of the lepton number into the electric charge yields quark quantum numbers from those of leptons. Hence the sum of electric and fermionic number is constant:

$$Q_\ell + B_\ell = Q_q + B_q .$$

It is then straightforward to construct the meson quantum numbers as $(\ell\bar{\ell})$ states from the known $(q\bar{q})$ configurations, both pseudoscalar and vector mesons.

In the case of baryons, the proton is always a final decay product of all baryons. The baryons cannot be constructed as $(\ell\ell\ell)$ states, because then, in analogy to (qqq), L would be equal to 3 and B = 0, but as $p\ell\bar{\ell}$ states giving total baryon number B = 1 and lepton number L zero. This assignment is in agreement with the meson theory of nuclear interaction, as we shall see.

The conservation of lepton and baryon numbers and charge are automatically built-in in this model, because p, e and ν are absolutely stable. The only dynamical processes are the pair production and exchange and rearrangement of constituents and these conserve Q, B and L. (see also Section V)

Strangeness

A physical interpretation of the mysterious internal quantum numbers, like isospin and strangeness, emerges from the model. As we have noted, the μ number is equal to the strangeness number S. Hence the number of all quantum numbers is reduced by 1: $S = N_{\mu+} - N_{\mu-}$.

The Isospin and its Physical Interpretation

The isotopic spin quantum number essentially counts the number of stable constituents (p, e and ν). In order to see this more precisely, we first define the third component of isospin and the isospin creation and annihilation operators

$$I_3 = \frac{1}{2}(N_p - N_{\bar{p}} + N_{e^+} - N_{e^-} + N_\nu - N_{\bar{\nu}})$$

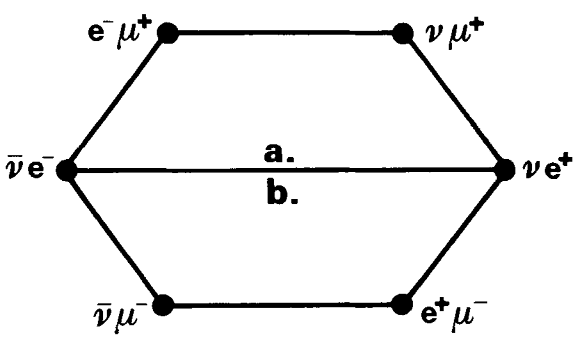
$$I_+ = \frac{1}{\sqrt{2}}(a_\nu^+ a_{e^-} + a_{e^-}^+ a_{\bar{\nu}}),\ I_- = (I_+)^\dagger \qquad (6)$$

The empirical Gell-Mann-Nishijima formula is now derived and automatically also built in the model:

$$Q = N_p - N_{\bar{p}} + N_{e^+} - N_{e^-} + N_{\mu^+} - N_{\mu^-} = I_3 + \frac{1}{2}(N_p - N_{\bar{p}} + S), \qquad (7)$$

because $\sum_\ell N_\ell = \sum_\ell N_{\bar{\ell}}$ for all states

$$(\text{i.e. } N_{e^+} + N_{\mu^+} + N_{\bar{\nu}} = N_{e^-} + N_{\mu^-} + N_\nu).$$



$$a = \frac{1}{\sqrt{2}}(\bar{\nu}\nu - e^+e^-),\ b = \frac{1}{\sqrt{6}}(\bar{\nu}\nu + e^+e^- - 2\mu^+\mu^-)$$

FIGURE 3. The Meson Octet

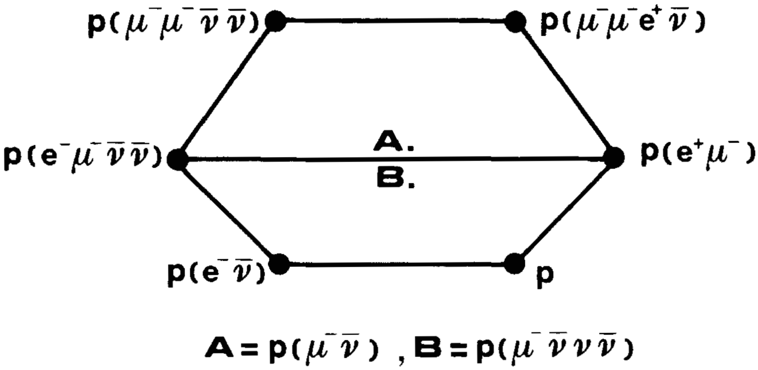


FIGURE 4. The Baryon Octet

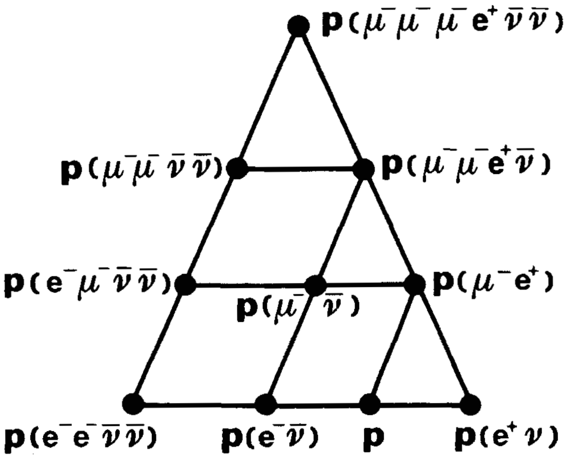


FIGURE 5. The Baryon Decouplet. The nearly linear mass formula of about the μ mass is a consequence of nearly zero-energy bound states in the magnetic potential well.

Figs. 3, 4 and 5 show the hadron multiplets in minimal realization.[15] We can of course add to each hadron lepton pairs ($\ell\bar{\ell}$) of the same species without changing the quantum numbers. For example, the physical proton can be thought of as having a π^0 cloud:

$$p_{physical} = p\left[\frac{1}{\sqrt{2}}(e^- e^+ - \nu\bar{\nu})\right] \qquad (8)$$

as can be seen by applying I_- to it or I_+ to the neutron state.

A full physical interpretation can be given to the concept of isospin as the quantum-mechanical exchange process of the lepton pair ($e^- \bar{\nu}$) between two systems, exactly like the exchange effects in H_2^+ molecule. To see this we go to the two-nucleon problem, where the notion of isospin has historically originated. The states of definite isospin are

$$pp, \frac{1}{\sqrt{2}}\ (pn + np),\quad nn\ (I = 1),\ \text{and}\ \frac{1}{\sqrt{2}}\ (pn - np)\quad (I = 0).$$

In the $I_3 = 0$ state, ($e\bar{\nu}$) is exchanged between the two protons and we have the symmetric ($I = 1$) and antisymmetric ($I = 0$) states with respect to the exchange, which are eigenstates of the total Hamiltonian. We could make a similar isospin triplet and singlet in atomic physics with

$$pp,\ \frac{1}{\sqrt{2}}\ (Hp + pH) \equiv H_2^-\ \text{sym.},\ H_2\ ;\ \frac{1}{\sqrt{2}}\ (Hp - pH) \equiv H_2^-\ \text{antisym.}$$

Here (p, H) is an isospin-doublet ($I_3 = +\frac{1}{2}$ and $-\frac{1}{2}$) and $Q = I_3 + \frac{1}{2}$. Also $I_+ = a_p a_H^+$. Similarly, if we look at two-pion states of definite isospin

$$|\pi^\pm\pi^\pm\rangle\ ,\ \frac{1}{\sqrt{2}}\{|\pi^\pm,\pi^0\rangle + |\pi^0,\pi^\pm\rangle\},\ \frac{1}{\sqrt{6}}\{2|\pi^0\pi^0\rangle + |\pi^+\pi^-\rangle + |\pi^-\pi^+\rangle\}$$

$$\frac{1}{\sqrt{2}}\{|\pi^\pm,\pi^0\rangle - |\pi^0,\pi^\pm\rangle\},\ \frac{1}{\sqrt{2}}\{|\pi^+\pi^-\rangle - |\pi^-\pi^+\rangle\}$$

$$\frac{1}{\sqrt{3}}\{|\pi^+,\pi^-\rangle + |\pi^-\pi^+\rangle - |\pi^0\pi^0\rangle\}\quad ,$$

or, pion-nucleon states of definite isospin

$$p\pi^+ ,\ \frac{1}{\sqrt{3}}\{2|p\pi^0\rangle + |n\pi^+\rangle\},\ \frac{1}{\sqrt{3}}\{2|n\pi^0\rangle + |p\pi^-\rangle\},\ n\pi^-$$

$$\frac{1}{\sqrt{3}}\{|p\pi^0\rangle - 2|n\pi^+\rangle\},\ \frac{1}{\sqrt{3}}\{-|n\pi^0\rangle + 2|p\pi^-\rangle\} \quad ,$$

we see that the isospin is identical to the symmetric and antisymmetric exchange, or rearrangement, of constituents. Isospin conservation is always used or tested in the reactions of two or more hadrons when stable constituents can be exchanged between the two hadrons,as between two atoms. It is convenient but not necessary to assign an isospin to individual hadrons, let alone to the constituents of hadrons, although the third component of isospin can be assigned to the constituents via the Gell-Mann-Nishijima formula. The conservation of the third component of isospin is equivalent to the conservation of the number of stable constituents, because the only processes occuring at the fundamental level, according to the present model, are the rearrangement of constituents when two hadrons interact, and pair production and annihilation of stable particles. The conservation of I or I^2 in strong interactions, on the other hand, is the conservation of symmetry properties of stable leptons ($e\bar{\nu}$) under exchange between the hadrons.

The physical intuitive meanings given to the abstract internal quantum numbers of hadrons is a significant feature of the present theory : The constituents no longer carry mysterious properties such as strangeness, isospin,charm, etc. The only charge is the electric charge.

Relation to Quark Assignements

The relation of our constituents to quark constituents is very simple. For mesons : $\ell\bar{\ell} \to q\bar{q}$, and for baryons : if we think of the proton as (uud), then our assignements become the same as the three quark assignement $q_1 q_2 q_3$ with additional definite $(q\bar{q})$ terms of the same species[2] (so-called $q\bar{q}$ sea terms). Such terms are introduced in the quark model anyway. Hence grouptheoretical results of the quark model remain intact in this model as well.

If we continue this correspondance (or shift) between quarks and leptons, then the next "excited" neutrino with the quantum numbers of ν_μ would correspond precisely to the so-called "charmed" quark and the next leptons τ and ν_τ to the other two new quarks, b and t. It is not known at present if ν_μ or ν_τ are massless or absolutely stable. According to the experimental limit so far, ν_μ is heavier than the electron! Because there is such a close symmetry between leptons and hypothetical quarks, it is most natural simply to identify them.

Nucleon Structure From Deep Inelastic Scattering Experiments

It is important to remark that from deep inelastic electron-nucleon scattering experiments one can infer two solutions for the charges of the constituents of the nucleon (assumed to be point-like at high energies) [16]. One solution gives for the proton constituents the charges +1, +1, -1, and for neutron constituents the charges +1, -1, 0. This is in agreement in our model with the physical proton being (pe^+e^-) and neutron being $(pe^-\bar{\nu})$. The second solution gives the fractional quark charges. Only the further assumption of additivity of the magnetic moments of quarks and equal additive quark masses then selects the second solution. However, in our dynamical physical bound state model, magnetic moments also have orbital contributions, and constituent masses are unequal. Magnetic moments must be calculated from the wave functions of the magnetic bound states. Thus it is not true, as generally advertised, that "deep inelastic experiments give "proof" of the existence of quarks".

V. STRONG AND WEAK INTERACTIONS

All strong interactions including nuclear forces are, according to the present theory, of magnetic type and are further determined by the composite structure of the hadrons. Specifically there are two fundamental processes at short distances when hadrons collide : i) Rearrangement of constituent stable particles, ii) pair production (or annihilation) of leptons (and subsequent rearrangement). It is possible to give diagrams for every strong process using i) and ii). The ideas of the old meson theory, the many models of meson exchanges or Regge-pole exchanges fit naturally and emerge as approximate schemes from this theory, as well as the ideas of the S-matrix theory and nuclear democracy: different rearrangements of constituents with real or virtual lepton pairs obviously imply that hadrons can be thought to be built of other hadrons. In particular, the meson cloud around the nucleon is an immediate approximation here, but not in the quark model. The sign of the neutron-proton mass difference is correctly explained by the theorem of positive binding energy of magnetic resonances. (see Section II)

Nuclear Model

We propose here a new model of the nucleus, which seems to combine two apparently contradictory features of the nucleus. On the one hand, the nucleus consists of closely packed large nucleons with an occupancy between 60 and 90%, or may even have a crystalline structure. On the other hand, the nucleons seem to be moving freely inside the nucleus, as the shell model or other Fermi gas models are implying. These two features are reconciled in the present theory as follows. The stable protons form the closed packing or even the crystalline skeleton of the nucleus. On top of it the stable lepton pairs $(e^- \bar{\nu})$ acting like a boson are hopping from one proton to another. When an $(e^- \bar{\nu})$ is attached to a proton, it then becomes a neutron. Thus moving $(e^- \bar{\nu})$'s will appear exactly as moving neutrons, or moving protons in the opposite direction. We can then study the motion of $(e^- \bar{\nu})$ pairs in the periodic potential of the lattice of protons.

Weak Interactions

The weak interactions of the β-decay type are due to barrier penetration, e.g. $n(pe^- \bar{\nu})$-decay or $\mu(e\nu\bar{\nu})$-decay. In fact, a theory of the neutron with an equation of type (1) - (2) correlates (in this approximation) the lifetime of the neutron, the n-p mass difference (which is positive and can be estimated as the excess magnetic energy of $(e^- \bar{\nu})$ bound to the proton) and the magnetic moment of the neutron [8]. Hence, indirectly, the Fermi constant G is related to the fine-structure constant α. All other decay modes of hadron can be understood as a (a) barrier penetration between two wells of the potential (see Fig. 1), (b) μ decay inside the hadron (suppressed by the Cabibbo angle as compared with the free decay) and (c) barrier penetration with or without μ decay. Different decay channels result in different rearrangements of the constituents. Finally, a weak scattering process such as $e\nu \to e\nu$ should be related to the anomalous magnetic moment of the neutrino. This possibility remains to be verified when we shall have more experimental data on the angular and energy dependence of this process.

VI. SOME FURTHER APPLICATIONS: K^0 PHYSICS AND CP VIOLATION

As an example of the intuitive value of the model we consider its application to the remarkable physics of the K^0 mesons.

According to Fig. 3, K^0 and $\bar{K}^0$ mesons are $(e^-\mu^+)$ and $(e^+\mu^-)$, respectively, i.e. the magnetic analogues of muonium and antimuonium. (Such states have also been called superpositronium $(e^+\mu^-)$ or supermuonium $(e^-\mu^+)$.) They are obviously charge conjugates of each other. If one of the states is produced, say $e^-\mu^+$, and we view μ^+ as $(e^+\nu\bar{\nu})$, then $(\nu\bar{\nu})$ pair can be exchanged, i.e. oscillate between e^- and e^+ in a magnetic potential as shown in Fig. 6. When $(\nu\bar{\nu})$ is attached to e^+ we have a $\bar{K}^0$, when it is attached to e^- we have a K^0. Under these circumstances, we know from general quantum mechanics that the observed eigenstates of the energy are the symmetric and antisymmetric combinations with respect to the $(\nu\bar{\nu})$ exchange, namely $K_S = K^0 + \bar{K}^0$, and $K_{AS} = K^0 - \bar{K}^0$, which are also eigenstates of CP. In fact, the problem is exactly the same quantum-mechanically as in the ammonium (NH_3) laser [17], where N oscillates between two positions in a potential as in Fig. 6. We therefore have the unambiguous prediction that the antisymmetric state is heavier than the symmetric one. In our case $m(K_L) > m(K_S)$. This is, to my knowledge, the first theory of the sign of the K_L-K_S mass difference. Moreover, the Dennison-Uhlenbeck mass formula [18] gives for the mass difference $\Delta m/m = 1/\pi A^2$, where A is the barrier penetration factor in the potential (Fig. 6). We do not know A, but we can obtain it from the decay rate Γ_s of K_S into $\pi^- + \pi^+$ $(e^-\bar{\nu} + e^+\nu)$, which uses the same potential barrier. This gives $\Delta m = \Gamma_s/2$. Experimentally we have for the K_L-K_S mass difference $\Delta m = 0.477\,\Gamma_s$.

The two decay modes of K_S are given by two ways of rearranging the constituents, namely: $\pi^-\pi^+$ and $\pi^0\pi^0$. However, K_L cannot decay in this way because of CP invariance. But an additional lepton pair production gives all the decay channels of K_L. The rate is then down by $(\pi\alpha)$ due to this pair production, which agrees with experiment.

Finally we discuss a mechanism of CP violation which occurs sofar only in the K^0 mesons. CP violation in our picture means a small violation of the symmetric and antisymmetric combinations K_S and K_{AS} introduced above. There is, in fact, a feature in the model, which brings an asymmetry. In the above discussion we have not made a distinction between ν_e and ν_μ. If we do make a distinction, then we have $(e^-\bar{\nu}_e\nu_\mu e^+)$ combination for $\bar{K}^0$ and $(e^-\bar{\nu}_\mu\nu_e e^+)$ combination for K^0. Hence an extra interaction must convert $\bar{\nu}_e\nu_\mu$ into $\bar{\nu}_\mu\nu_e$, which provides a further asymmetry between K_1 and K_2 leading to K_L and K_S. We can further predict that CP violation should also occur in the neutral mesons built from ($e^-\tau^+$ and $e^+\tau^-$) and ($\mu^-\tau^+$ and $\mu^+\tau^-$).

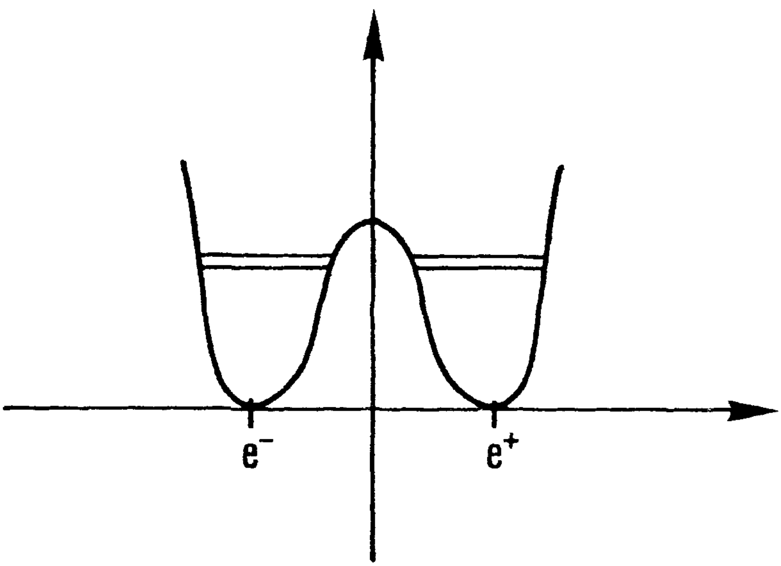


FIGURE 6. The effective magnetic potential barrier for ($\nu\bar{\nu}$) - exchange between e^- and e^+, and the two symmetric and antisymmetric states in the K^0 - K^0 system.

VII. CONCLUSION

High-energy physics according to the present theory can be considered as an extension of atomic and molecular physics. The Coulomb forces being replaced by the short-ranged strong magnetic forces. The only additional particle not present in atomic physics is the neutrino, which is in fact a limiting case of the electron. There is then a welcome continuity and simplicity in the physics, which was perhaps lost by the abstract concepts and free inventiveness of particle physics. No new particles, or no new interactions or forces are introduced [19] except the stable ones and the electromagnetic field. In this sense it is a truly already-unified theory with one coupling constant e. The only parameter so far, in principle, is the neutrino magnetic moment. All other "particles" are transitory; they come as resonances and eventually decay into the absolutely stable particles. The division of forces in nature into strong, weak and elementary was a temporary one; there is no need for such a division.

Although much detailed quantitative work must be done, and is being done, we have shown that, conceptually and logically, it is possible to understand the world of fundamental particles and their interactions from the very simple framework of stable particles and stable electromagnetic forces. Our guiding principle has been the same as that of Lord Kelvin under similar circumstances: "I want to understand light as well as I can, without introducing things that we can understand even less of."

There are a number of very simple and fundamental quantities in particle physics, such as (i) absolute masses of hadrons, e.g. the neutron-proton mass difference, (ii) the dipole form factors of the nucleons, (iii) scattering lengths, (iv) magnetic moments of hadrons, which have been passed over as uncalculable by the QCD perturbation theory, for example. The present framework seems to be particularly well suited to calculate these basic dynamical parameters. And this will further test and determine the direction of the theory.

APPENDIX I. MODELS OF MAGNETIC INTERACTIONS

Model 1. Pauli Equation

For a nonrelativistic charge e_1 moving in the vector potential $\vec{A} = \mu_2(\vec{\sigma}\times\vec{r}/r^3)$ of another charge e_2 with magnetic moment μ_2 the Pauli equation

$$\left[\frac{1}{2m}(\vec{p}-e_1\vec{A})^2+e_1e_2/r \right]\Psi=E\Psi$$

for stationary states, with

$$i\frac{e\hbar}{mc}\vec{A}.\nabla = -\frac{e_1\mu_2}{mcr^2}\vec{\sigma}.\vec{L} \ , \quad A^2=\mu_2^2/r^4$$

reduces to

$$\left\{-\frac{\hbar^2}{2m}D_r + \frac{1}{2mr^2}L^2-2\frac{e_1\mu_2}{m\hbar cr^3}\vec{S}.\vec{L} + \frac{e_1^2\mu_2^2}{2mc^2r^4} + \frac{e_1e_2}{r} - E\right\}\Psi = 0$$

where

$$D_r = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}).$$

Introducing $\vec{J} = \vec{L} + \vec{S}$ and expanding in spherical harmonics to separate the angular variables, we obtain the radial equation

$$\left\{-\frac{\hbar^2}{2m}D_r+\frac{\ell(\ell+1)\hbar^2}{2mr^2} - \frac{e_1\mu_2}{mc\ r^3}\left[J(J+1)-\ell(\ell+1)-\frac{3}{4}\right]\hbar^2+\frac{e_1^2\mu_2^2}{2mc^2r^4} + \frac{e_1e_2}{r} - E\right\} U_{\ell J}(r) = 0 \cdot$$

This is essentially the eq.(10) with (2) of the text. It is important not to neglect in this equation the A^2-term, contrary to the usual perturbative calculations, because otherwise the $1/r^3$- term would dominate as $r \to 0$, and the Hamiltonian in an exact treatment would not be well-defined and essentially selfadjoint. The $1/r^4$ - term which is always repulsive makes the problem well-defined and soluble.

For a given sign of μ_2, say $\mu_2 > 0$, the $1/r^3$-term in (AI.1) is negative only for $\ell = J+\frac{1}{2}$ if $e_1 < 0$, or $\ell = J-\frac{1}{2}$ if $e_1 > 0$. We expect therefore resonances for $(J=\frac{1}{2}, \ell=1), (J=3/2, \ell=2), \ldots$ for $e_1 < 0$, and $(J=3/2, \ell=1), (J=5/2, \ell=2), \ldots$ for $e_1 > 0$.

Model 2. Klein-Gordon Charged Particle in the Field of a Magnetic Moment

The Hamiltonian is given by

$$H = \left[(\vec{P}-e\vec{A})^2 + m^2\right]^{\frac{1}{2}} + A_o.$$

In the absence of A_0 and by proceeding exactly as in Model 1, we are lead to the same final equation (AI.2), except that the eigenvalue is now

$$(E_{rel}^2 - m^2)/2m, \quad E_{rel} = (2mE^2 + m^2)^{\frac{1}{2}},$$

where E is the eigenvalue of eq.(AI.2), i.e. the nonrelativistic energy.

Model 3. The Dirac Particle

We start from the most general coupled radial Dirac equations containing a scalar potential V_s, an electric (Coulomb) potential V_e and a magnetic potential V_m :

$$\frac{df}{dr} = \frac{\kappa-1}{r} f + (m + V_s - E)g + V_e g + V_m f$$

$$\frac{dg}{dr} = -\frac{\kappa+1}{r} g + (m + V_s + E)f - V_e f - V_m g$$

From these equations we obtain a second order Sturm-Liouville eigenvalue equation[10])

$$\Psi'' + (E^2 - m^2 - V_{eff})\Psi = 0 ,$$

where

$$V_{eff}^{(1)} = \frac{\kappa(\kappa+1)}{r^2} + 2EV_e - V_e^2 + V_s^2 + 2m\, V_s + V_m^2 + 2\,\frac{\kappa V_m}{r} - V_m'$$

$$+ \frac{1}{2}\,\frac{V_e'' - V_s'' + 2(V_s' - V_e')\left(\frac{\kappa}{r} + V_m\right)}{(m + E + V_s - V_e)} + \frac{3}{4}\,\frac{(V_s' - V_e')^2}{(m + E + V_s - V_e)^2}$$

Note that the potential V_{eff} is both energy(E) and angular momentum dependent. It must be evaluated at each E and κ and the the eigenvalue problem for bound states and resonances must be solved at these values of E and κ

For the normal Dirac particle in the Coulomb field we have $V_s=0$ and $V_m=0$, and the effective potential is given by

$$V_{eff} = \frac{\kappa(\kappa+1)-\alpha^2}{r^2} + \varepsilon\frac{2E\alpha}{r}+\varepsilon\frac{\alpha(\kappa+1)}{r^2[m+E)r-\varepsilon\alpha]} + \frac{\frac{3}{4}\,\alpha^2}{r^2[(m+E)r-\varepsilon\alpha]^2} ,$$

where $\varepsilon = \text{sign}\,(e_1e_2)$. Here the particle itself carries a normal magnetic moment with g=2, and the other particles is a fixed charge, so that this model is dual to the models 1. and 2., where the magnetic moment was fixed at the center.

Model 4. Spin ½ Particle with anomalous Magnetic Moment in the Coulomb Field

This case leads to a surprising new additional effect. In the general effective potential (AI.7) we have now

$$V_e = e_1 e_2 / r \,, \quad V_m = a e_1 e_2 / 2mr^2 ,$$

where a is the anomalous magnetic moment in addition to the normal magnetic moment g=2. This follows from the Pauli coupling of the relativistic particle.[5] The efeective potentila now becomes (for $\varepsilon = -1$)

$$V_{eff} = \frac{\kappa(\kappa+1)-\alpha^2}{y^2} - \frac{\alpha^3}{2\pi}\frac{E}{m}\frac{1}{y} + \frac{1}{y^2}\left[\frac{-(\kappa+1)}{h(y)} + \frac{3}{4}\frac{1}{h^2(y)}\right]$$

$$+ \frac{1}{y^3}\left[-2(\kappa+1) + \frac{1}{h(y)}\right] + \frac{1}{y^4} \,,$$

where

$$y = 2a\frac{m}{a\alpha}r, \quad \Lambda^2 = a^2 \frac{\alpha^2}{4m^2}(E^2 - m_r^2), \quad \text{and } h(y) = 1 + \frac{\alpha}{4\pi}\frac{E+m/2}{m} y.$$

This potential has at most five real zeros (see Fig.1), giving in general the three potential wells. Depending on E and κ, of course, not all the wells will be pronounced at the same time.

Model 5. Piron-Reuse Relativistic Particle

Here one uses a covariant wave equation of the form

$$i\hbar \frac{\partial \Psi}{\partial \tau} = K\Psi \quad ,$$

where τ is an invariant evolution parameter, and

$$K = \frac{1}{2M}(P_\mu - eA_\mu)^2 - g_1\frac{\mu_o}{M}(P_\mu - eA_\mu)\tilde{F}^{\mu\nu}W_\nu + g_2^2\frac{\mu_o^2}{8M}F_{\mu\nu}\eta^\nu F^\mu_\rho \eta^\rho$$

$$-g_3 \mu_o \eta^\mu \tilde{F}_{\mu\nu} W^\nu$$

Here g_1, g_2, g_3 are constants (the g-factor of the particle being $g = 2g_1 + g_3$), n^μ is a timelike unit vector and W^μ the relativistic spin fourvector. In the Coulomb field the stationary radial equation coincides exactly with eq.(1)

of the text.

Model 6. Inclusion of Recoil and Spin-Spin Terms

Spin-spin interactions can be taken into account as follows. In models 1. and 2. we must add to the Hamiltonian the energy $-\vec{\mu}_1 . \vec{B}_2$ of the magnetic moment of particle 1 in the field $\vec{B}_2$ produced by particle 2. This gives the additional term

$$\vec{\mu}_1 . \vec{B} = -\mu_1\mu_2 \frac{3\vec{\sigma}_1 . \hat{r}\vec{\sigma}_2 . \hat{r} - \vec{\sigma}_1 . \vec{\sigma}_2}{r^3} + \frac{8\pi}{3} \mu_1\mu_2 \vec{\sigma}_1 . \vec{\sigma}_2 \, \delta^3(r)$$

In models 3. and 4. we write the Dirac equation in the magnetic potential $A = \mu_2\vec{\sigma}_2 x\hat{r}/r^2$ of the second particle, i.e.

$$H = \vec{\alpha} . (p_1 - e_1\mu_2 \frac{\vec{\sigma}_2 x\vec{r}}{r^3}) + \frac{e_1 e_2}{r} + \beta_1 m + a \frac{e_1}{2m} (\frac{e_2}{r^2} \, i\beta_1 \frac{\vec{\alpha} . \vec{r}}{r} - \beta_1 \vec{\sigma}_1 . \vec{B}_2),$$

where the last term is as in the previous equation. This interesting Hamiltonian which to our knowledge has not been studied before, has now been completely separated and the results will be presented shortly.

Model 5. has also been extended to two-particle systems. We have also studied relativistic two-body problems in the socalled one-time formalism [7]. To this list of models one must add the various potentials obtained from the Bethe-Salpeter type of equations which all now should be treated nonperturbatively.

APPENDIX II. ANALYTIC PROOF OF THE EXISTENCE OF HIGH ENERGY NARROW RESONANCES

The importance in quantum mechanics of exactly soluble bound state eigenvalue problems is wellknown. These problems involve, in our terminology, electric or scalar potentials. It turns out that a class of magnetic potentials are also exactly soluble, this time as an eigenvalue problem for narrow resonances of complex energy. Since all hadrons and leptons (except p, e, ν) are unstable, these soluble cases will be as basic to our theory as the Coulomb case is for atomic theory, or the oscillator problem for nuclear and molecular theory. The result is embodied in the following[20]
Theorem : The reduced eigenvalue problem, eq.(1), with a potential

$$V(y) = \frac{v_2}{y^2} - \frac{2(M+1)}{y^3} + \frac{1}{y^4}$$

is exactly soluble in the space of functions defined by

$$u(r=0) = 0, \quad u(r) \xrightarrow[r\to\infty]{} e^{i\lambda r}.$$

The resonance quantization condition is given by

$$\det \Delta = 0,$$

where the (M+1)x(M+1) matrix Δ is , with $D = M^2+M+2i\lambda-v_2$,

$$\Delta = \begin{vmatrix} D & 2 & 0 & & \\ -2i\lambda M & D-2M & 4 & & \\ 0 & -2i\lambda(M-1) & D+2(1-2M) & 6 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -2i\lambda & & D+M(1-M) \end{vmatrix}$$

There are, for a given M, M+1 complex eigenvalues.

In the spin-orbit potentials, M has precisely the meaning of the Dirac quantum number κ (or ℓ in nonrelativistic case). For M=0, we have one purely imaginary eigenvalue $\lambda = iv_2/2$ For M=1, we find from the 2x2 determinant

$$\lambda = -i\left(\frac{v_2-2}{2}\right) \pm \left(2(v_2-2)\right)^{\frac{1}{2}} .$$

If $v_2=2=\kappa(\kappa+1)$, the eigenvalue is zero, i.e. zero-energy solution in agreement with the exact direct solution[10].

APPENDIX III. NEUTRINO BOUND STATES

We evaluate the limit of Model 4., App.I, when $e_1 \to 0$, $m \to 0$, in such a way that $ae_1/2m = \mu$ = anomalous magnetic moment of the neutrino is different from zero. The effective potential reduces to (with formfactor F_m included)

$$V^{(1)}_{eff}(y) = \frac{\kappa(\kappa+1)}{y^2} + \varepsilon\frac{2(\kappa+1)}{y^3} F_m + \frac{1}{y^4} F_m^2$$

where $y = r/|e\mu|$, $\varepsilon = \mathrm{sign}(e\mu)$ and eigenvalue $\lambda^2=|e\mu|^2E^2$. This equation has an exact zero-energy solution . The normalizable eigenfunctions are such that two of the four components must vanish, i.e.

$$\begin{pmatrix} ig \\ 0 \end{pmatrix} \text{ for } \varepsilon=-1,\ \kappa=+1, \text{ and } \begin{pmatrix} o \\ f \end{pmatrix} \text{ for } \varepsilon=+1,\ \kappa=-1.$$

In addition, spin-spin term must be added to this solution.

APPENDIX IV. BARRIER PENETRATION

If we approximate the potential in eq.(2) by two square wells, one positive and one negative, i.e.

$$V(r) = \begin{cases} -V_1 & \text{for} \quad o \leqslant r < r_1 \\ +V_o & \text{for} \quad r_1 \leqslant r < R \\ 0 & \text{for} \quad r \geqslant R, \end{cases} \quad .$$

then the phase shift corresponding to eq.(1) can be calculated exactly. Note that the angular momentum barrier is in V_0. The result is

$$\delta = -\lambda R + \tan^{-1}\left\{ \frac{\lambda}{a} \frac{\tan aR+\delta}{1+\gamma\tanh aR} \right\}$$

where $\quad a^2 = V_o^2 - \lambda^2, \quad K^2 = V_1^2 + \lambda^2$

and

$$\gamma = \frac{\frac{a}{K}\tan Kr_1 - \tanh ar_1}{1-\frac{a}{K}\tan Kr_1 \tanh ar_1}$$

As a function of energy(i.e.λ), the phase shift indeed jumps suddenly at $E=E_r$ by about π as shown in Fig.2. We then calculate the partial wave cross section $\sigma_\ell = \frac{1}{\lambda}\sin^2\delta_\ell$ which has two zeros very close to each other and near $E=E_r$. If we add this partial wave cross section to the backgroud of all others $\sum\sigma_\ell$ we obtain the effect of barrier penetration on the total cross section which is shown in the lower curve in Fig. 2.

APPENDIX V. ANALYTICITY IN ANGULAR MOMENTUM AND REGGE POLES

It is known that the analytic properties of the scattering amplitude in the left half angular momentum plane, Re $\ell < -\frac{1}{2}$, depend essentially on the behavior of the potential at short distances. Even though the relativistic effective potential is energy dependent we can study analyticity in ℓ for each fixed E. For potential of the type

$$V = \frac{A}{r^4} + \frac{B}{r^3} + \frac{C}{r^2} + \int_0^\infty \sigma(x) \frac{e^{-xr}}{r} \, dx$$

Predazzi and Regge[22] have shown that the regular solution is an entire function of both of the coefficients B and C, but not of A at A=0. The latter is because if A is negative we have an attractive singular case. Hence there is no perturbative expansion of the regular solution in the coupling constant A. This is in agreement with our statement that the resonance solution cannot be obtained in perturbation theory. The physical reason of the analyticity in C, hence in $\ell(\ell+1)$, or in $(\ell+\frac{1}{2})$, is that for small r, the term A/r^4 dominates which is independent of ℓ. It follows further that the S-Matrix for fixed energy is meromorphic in ℓ, and therefore the scattering amplitude can be expressed as a sum Regge poles only, with no backgroud integral or Regge cut terms,

$$f(E,\theta) = 2\pi \sum_n (2\alpha_n+1)\beta_n P_{\alpha_n}(\cos\theta) \frac{e^{-i\pi(\alpha_n+1)}}{in\pi\alpha_n}$$

The backgroud near a resonance pole will come from the contribution of all other far away poles. (see also Fig.2 and App.IV.) We have here a realization of the principle of maximal analyticity in angular momentum.[23]

In addition the magnetic potential result in a differential cross section which increases as $\log^2(s/s_0)$ in agreement with the high energy two-body cross sections.[24]

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