ANALYTICAL MODELS

Ordinary Differential Equations

linear differential equations

$$\frac{d^2u}{dx^2} - x\frac{du}{dx} + u = 0.$$

systems of differential equations

$$R_1(C_1 + C_2)\frac{dV_1}{dt} + V_1 = R_1C_2\frac{dV_2}{dt} + V_{DD}$$
$$R_2(C_2 + C_3)\frac{dV_2}{dt} + V_2 = R_2C_2\frac{dV_1}{dt}$$

laplace transforms

$$F(s) = \int_0^\infty e^{-st} f(t)\,dt$$
$$H(s) = \frac{1}{(s+\alpha)(s+\beta)}.$$

perturbation expansions

$$\frac{d^2y}{dx^2} = -\epsilon\frac{dy}{dx} - 1,$$
$$y(\epsilon) = y_0(\epsilon) + \epsilon y_1(\epsilon) + \epsilon^2 y_2(\epsilon) + O(\epsilon^3)$$

Difference Equations

discrete time equations

$$d_{n+2} - d_{n+1} - d_n = 0$$

z-transforms

$$X(z) = Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$
$$H(z) = \frac{b_1^2}{1 - (2 - k_1(k_2 + k_0^2))z^{-1} + (1 - k_1k_0)z^{-2}}$$

Partial Differential Equations

hyperbolic equations / waves

$$\nabla^2\varphi = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$$

parabolic equations / diffusion

$$\nabla^2\varphi = \frac{1}{D}\frac{\partial\varphi}{\partial t}$$

elliptic equations / boundary values

$$\nabla^2\varphi = \rho$$

separation of variables

$$u(\mathbf{r}, t) = A(\mathbf{r})T(t).$$
$$A(r, \theta) = R(r)\Theta(\theta),$$

Variational Principles

variational calculus

$$\mathcal{I} = \int_{x_1}^{x_2} f[y(x), y(x), x]\,dx$$
$$y(x, \alpha) = y(x) + \alpha\eta(x)$$

euler's equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0$$

constraints and lagrange multipliers

$$\int_{x_1}^{x_2} g[y(x), y(x), x]\,dx = C$$
$$h = f + \lambda g$$

Random Systems

random variables

$$\int_{-\infty}^{\infty} f(x)\,p(x)\,dx$$

joint distributions

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\,p(x, y)\,dx\,dy$$

bayes' rule

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

characteristic functions

$$\int_{-\infty}^{\infty} e^{ikx} p(x)\,dx$$

stochastic processes

$$p(x_t|x_{t-\tau_1}, x_{t-\tau_2}, \ldots, x_{t-\tau_N})$$

random number generators

$$x_{n+1} = ax_n + b \pmod c$$
$$x_n = x_{n-1} + x_{n-4} + x_{n-8} + x_{n-12}$$

NUMERICAL MODELS

Finite Differences ODEs

euler's method

$$y(x + h) = y(x) + h\,f(x, y(x))$$

runga-kutta methods

$$k_1 = h\,f(x, y(x))$$
$$k_2 = h\,f\left(x + \frac{h}{2}, y(x) + \frac{k_1}{2}\right)$$
$$k_3 = h\,f\left(x + \frac{h}{2}, y(x) + \frac{k_2}{2}\right)$$
$$k_4 = h\,f(x + h,\ y(x) + k_3)$$
$$y(x + h) = y(x) + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O(h^5)$$

predictor-corrector methods

$$y_4(x + h) = y(x) + \frac{h}{24}(55f[x, y(x)] - 59f[x - h, y(x - h)] + 37f[x - 2h, y(x - 2h)] - 9f[x - 3h, y(x - 3h)])$$
$$y_c(x + h) = y(x) + \frac{h}{24}(9f[x + h, y_4(x + h)] + 19f[x, y(x)] - 5f[x - h, y(x - h)] + f[x - 2h, y(x - 2h)])\ .$$

Finite Differences PDEs

leapfrog method

$$u_j^{n+1} = u_j^{n-1} - \frac{u\Delta t}{\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right)$$

crank-nicolson method

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{D}{2(\Delta x)^2}\left[\left(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}\right) + \left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)\right]$$

jacobi's method

$$u_{i,k}^{n+1} = u_{i,k}^n + \frac{\Delta t}{(\Delta x)^2}\left(u_{i+1,k}^n + u_{i-1,k}^n + u_{i,k+1}^n + u_{i,k-1}^n + u_{i,k}^n(-4)\right) - \Delta t\,\rho_{i,k}$$

successive over-relaxation

$$u_{i,k}^{n+1} = (1 - \omega)u_{i,k}^n + \frac{\omega}{4}\left(u_{i+1,k}^n + u_{i-1,k}^{n+1} + u_{i,k+1}^n + u_{i,k-1}^{n+1} - \frac{\alpha(\Delta x)^2}{\epsilon_0}\rho_{i,k}\right)$$

Finite Elements

expansion in basis functions

$$u(\vec{x}, t) \approx \sum_n a_n(t)\phi_n(\vec{x})$$

weighted residuals

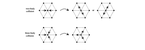
$$R(\vec{x}, t) = D[\tilde{u}(\vec{x}, t)] - f(\vec{x}, t)$$
$$\int R(\vec{x})w_n(\vec{x})\,d\vec{x} = 0$$

rayleigh-ritz variational methods

$$\delta\mathcal{I} = \delta\int F[u(\vec{x}, t)]\,d\vec{x} = 0$$
$$\frac{\partial\mathcal{I}}{\partial a_n} = 0$$

Cellular Automata & Lattice Gases

lattice gases & fluids



cellular automata & computation



OBSERVATIONAL MODELS

Function Fitting

bayes' rule

$$\max_{\vec{x}}\ p(\vec{a}|d, m) = \max_{\vec{x}}\ \frac{p(\vec{d}|\vec{a}, m)\,p(\vec{a}|m)}{p(\vec{d}|m)} = \max_{\vec{x}}\ \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

maximum likelihood

$$\max_{\vec{a}}\ p(\vec{a}|\vec{a}) = \max_{\vec{a}}\ p(\vec{d}|\vec{a})$$

least squares

$$\min\sum_{n=1}^{N}[y_n - y(x_n, \vec{a})]^2$$

gradient descent

$$\chi^2(\vec{a}) = \sum_{n=1}^{N}\left(\frac{y_n - y(x_n, \vec{a})}{\sigma_n}\right)^2$$
$$\vec{a}_{new} = \vec{a}_{old} - \alpha\nabla\chi^2(\vec{a}_{old})$$

levenberg-marquardt method

$$M_{ii} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial a_i^2}(1 + \lambda)$$
$$M_{ij} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial a_i\partial a_j} \qquad (i \neq j)$$
$$\delta\vec{a} = -\mathbf{M}^{-1} \cdot \nabla\chi^2$$

Architectures

polynomials

$$y(x) = \sum_{n=0}^{N} a_n x^n$$

padé approximants

$$y(x) = \frac{\sum_{n=0}^{M} a_n x^n}{1 + \sum_{n=1}^{N} b_n x^n}$$

splines

$$\vec{x}_i(t) = \vec{c}_{i-3}\varphi_{i-3}(t) + \vec{c}_{i-2}\varphi_{i-2}(t) + \vec{c}_{i-1}\varphi_{i-1}(t) + \vec{c}_i\varphi_i(t) \qquad (t_i \leq t < t_{i+1})$$

orthogonal functions

$$y(\vec{x}) = \sum_{n=1}^{M} a_n f_n(\vec{x})$$
$$\int_{-\infty}^{\infty} f_i(\vec{x}) f_j(\vec{x})\,d\vec{x} = \delta_{ij}$$

radial basis functions

$$y = \sum_{n=1}^{N} a_n f(|\vec{x} - \vec{c}_i|)$$

neural networks

$$X_j = g(\sum_i w_{ji} x_i)$$
$$Y_i = g(\sum_j W_{ij} X_j)$$

Transforms

orthogonal transforms

$$\mathbf{M}^T \cdot \mathbf{M} = \mathbf{I}$$

discrete fourier transform

$$X_f = \frac{1}{\sqrt{N}}\sum_{n=0}^{N} e^{2\pi i f n/N} x_n$$

discrete wavelet transform

$$y_{low}[n] = \sum_{k=-\infty}^{\infty} x[k]h[2n - k]$$
$$y_{high}[n] = \sum_{k=-\infty}^{\infty} x[k]g[2n - k]$$

principal component analysis

$$\mathbf{C}_{\vec{y}} \equiv \mathbf{M} \cdot \mathbf{C}_{\vec{x}} \cdot \mathbf{M}^T$$

Optimization & Search

downhill simplex method

momentum

simulated annealing

genetic algorithms

Clustering & Density Estimation

histograms and k-D trees



fitting densities

$$P(x) = x + \sum_{n=1}^{M} a_n \sin(n\pi x)$$

kernel density estimation

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

mixture models

$$p(\vec{x}) = \sum_{n=1}^{M} p(\vec{x}|c_m)\,p(c_m)$$

cluster-weighted models

$$p(y, \vec{x}) = \sum_{n=1}^{M} p(y|\vec{x}, c_m)\,p(\vec{x}|c_m)\,p(c_m)$$

Filtering & State Estimation

matched filters

$$f(\omega) = A\,e^*(t - w)$$

wiener filters

$$F(\omega) = \frac{C_{x+s+n}(\omega)e^{i\omega t_0}}{C_{x+s+n}(\omega)}$$

kalman filters

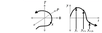
$$\vec{x}_{i|i} = \vec{x}_{i|i-1} + \mathbf{K}_i \cdot (\vec{y}_i - \vec{y}_{i|i-1})$$

hidden markov models



Nonlinear Time Series

state-space reconstruction



dimension measurement

$$D_q \equiv \lim_{\epsilon\to 0}\frac{1}{q-1}\frac{\log\sum_i p_i(\epsilon, l)^q}{\log\frac{1}{\epsilon}}$$

lyapunov exponents

$$\lambda = \lim_{t\to\infty}\lim_{|\delta Z_0|\to 0}\frac{1}{t}\ln\frac{|\delta Z(t)|}{|\delta Z_0|}.$$

entropies

$$H(x, N) = \sum_{n_1, \ldots, n_d}^{N} p(i_0, i_1, \ldots) \log_2 p(i_0, i_1, \ldots)$$