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COUNTER CULTURE: TOWARDS A HISTORY OF GREEK NUMERACY

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1. INTRODUCTION

This is a programmatic article. The field offered for research is extremely multidisciplinary, and no author can hope to be truly an expert in all of the issues touched. It is the author's hope that the inherent value of seeing this multidisciplinary field *as a field* outweighs the pitfalls that accompany such programs.

The article makes four claims:

- There is a need for a *history of numeracy*, alongside and complementing the field of the history of literacy.
- This history of numeracy should be seen as part of *cognitive history*: the study of culturally specific practices, in which universal human cognitive abilities are assembled together and implemented with the aid of specific tools and technologies.
- A certain assemblage of numerical practices, which I call *counter culture*, permeates Greek culture: here is a case where cognitive history plays an important role in cultural, political and economic history.
- The numerical practices mentioned above were typical not of numerical record, but of numerical manipulation. Thus Greek culture is characterized by a divide between numerical record and numerical manipulation. This divide, in turn, may have significant historical consequences.

The claims above are, as stated, opaque. The main purpose of this article is to unpack and clarify them. Following further preliminary notes below, I move on in Section 2, "Counter culture", to discuss some aspects of Greek numerical practice in (2.1) calculation itself, (2.2) the economy, (2.3) politics, and (2.4) the symbolic domain. Section 3, "Towards a history of Greek numeracy", tentatively offers directions for an interpretation of the evidence discussed in Section 2.[1]

1.1. *A Preliminary Note: The History of Numeracy*

The study of writing and its consequences is central to modern research in the humanities, especially since Goody's studies from the 1960s onwards, and the controversy surrounding them.[2] In the fields of classics and ancient history, literacy studies have a long tradition. (Arguably the most important work in the study of literacy taken as a whole is that of M. Parry, *The making of Homeric verse* (Oxford, 1971) — a collection of articles published from the 1920s onwards — where the theory of oral formulaic epic poetry was developed with Homer as its focus.) More recently, the

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field of ancient literacy has been augmented under the influence of the wider interest in literacy, and the work of W. V. Harris, *Ancient literacy* (Cambridge, Mass., 1989), marked the beginning of a growing body of research in ancient reading and writing.[3] Many research questions are raised, among which one may mention: the specifically oral nature of archaic Greek society and literature prior to the introduction of the alphabet; the history of the alphabet and its different uses; and the spread of literacy (in its various forms) in the classical Mediterranean.

Numeracy is far less researched. Hollis, the catalogue of the Harvard libraries, had, early in the year 2000, 21 entries with 'numeracy' as a keyword (not necessarily in the title of the work). The equivalent search for 'literacy' broke down, since the system could not handle more than 1000 entries simultaneously. Narrowing down to 'numeracy+history' and 'literacy+history', the numbers became 2 and 306, respectively.[4] There may be many reasons for this discrepancy. Perhaps many scholars in the humanities simply feel more at ease discussing literacy, their field of proficiency *par excellence*, than they feel about numeracy — which is typically the area they found less appealing during their own schooling. More significant, the very concept of 'numeracy' is far less established than that of 'literacy'. As I write, my software rolls a red squiggle, carpet-like, below the word 'numeracy' (it has no difficulties with 'literacy'). The issue is not merely lexical: at the conceptual level, literacy is much more clearly defined than is numeracy. For instance, I quote from a recent Unesco publication on measuring educational achievement in the three main domains of 'life skills', 'literacy', and 'numeracy'. Putting 'life skills' aside, the author says under *literacy* that "items concerning reading skills fall into two general categories essential for acquiring further skills: reading/reading comprehension and writing/writing comprehension", and under *numeracy* that "this domain examines the child's ability to perform simple arithmetic as well as solve exercises. It is important because it reflects his/her capacity for logical thinking and abstraction which is vital for everyday life".[5] Literacy is clearly defined as the ability to use a precisely given practice — writing. Numeracy is defined not by reference to a practice, but by a loosely given subject matter. Its significance is seen in reflecting some other, deeper abilities ("logical thinking and abstraction"). This, then, may be the prevalent image. Literacy has to do with writing, which is a clearly defined, simple practice: it therefore has a clearly defined, simple history. Numeracy, on the other hand, has to do with deep abilities related to some a-historical, mathematical and logical reality: it therefore has no clear history. Both sides of this image are false.

Note to begin with that the study of literacy itself has moved away from the simple image of writing as the alpha and omega of literacy. The cognitive impact of writing, in Goody's later, refined model, is due not to the use of the system of writing itself, but to its subsequent implications in wider cultural practices.[6] Further, current studies concentrate on the differential nature of writing: its varying uses by different practitioners, in different contexts.[7] Finally, let us remember that even Goody's original, more technology-centred approach, paid much attention to the different *kinds* of writing systems, contrasting Greek alphabetic script with Mesopotamian syllabic and Egyptian and Chinese ideograms. In short, the simplicity of 'literacy' is deceptive: there is not one practice, but many, and these practices can be used in many different ways. The history of literacy is the history of literacies.

At the same time, note also that numeracy is not — as implicit in the Unesco formulation — independent of specific cultural practices (with Unesco, numeracy has to do with 'arithmetic', understood presumably at an a-historical, purely mathematical level). Modern arithmetic — until very recently — was simply the system of Arabic numerals: invented at the end of the first millennium and made truly common in the West and elsewhere only from the sixteenth century onwards.[8] This system makes use of a-historical, purely mathematical features of numbers to allow their easy record and manipulation with pen and paper. Historically, there were other numerical systems available, and their history should address their differential uses by various practitioners and in various contexts — all exactly analogous to the study of literacy, so that there is no inherent difference between the history of numeracy and the history of literacy. Indeed, as hinted already for Arabic numerals, the two are intertwined. Arabic numerals are, among other things, a tool for bringing numerical practices into contact with verbal practices: they allow arithmetic to benefit from the practices of pen and paper, that is the typical writing practices of modernity. The most useful level of analysis, therefore, is not that of "history of numeracy" alone, or of "history of literacy" alone, but what I shall call "cognitive history": the study of the ways in which basic human cognitive skills are brought together in cultural practices, often aided by special tools.

In this article, however, I shall to a large extent isolate numeracy from literacy, for reasons which will become obvious below. In the Greek world, it may be analytically useful to separate numeracy from literacy. But even aside from that, one must stress that the example of Arabic numerals is in a sense historically misleading. With Arabic numerals, numbers appear as secondary to writing, benefiting from tools that were largely invented to record verbal symbols and not numerical symbols. In broad historical perspective, this is the exception and not the rule. The rule is that, across cultures, and especially in early cultures, the record and manipulation of numerical symbols precede and predominate over the record and manipulation of verbal symbols. Clay tablets, whether in Mesopotamia or in Crete, contain almost nothing but inventories, mostly numerical in character; the same is largely true of the papyri findings from Hellenistic and Roman Egypt. The Inca Quipu, the main information tool in the pre-Columbian Andes, may have been predominantly a record of numerical data — or of data understood on the model of the numerical.[9] And the most promising theory of the emergence of writing — that of D. Schmandt-Besserat, *Before writing*[10] — argues that writing emerged in Mesopotamia from previous tools of *numerical* record (we shall need to return below to the work of Schmandt-Besserat). In short, in early cultures, numeracy drives literacy rather than the other way around. Nor should one underestimate the extent to which this is true in the contemporary world: today's *New York Times* has its verbal parts, but the Business, Sports and Weather sections, to name three prominent examples, are as numerical as a Mycenian clay tablet. Numeracy, even today, is not yet a further skill, a footnote to writing: it is one of writing's essential features.

1.2. *A Preliminary Note: "Counter Culture"*

It is time to apologize for the pun at the title and to explain it. "Counters" — small, easily moveable tokens — are the subject of this article. I shall argue that one can detect, in the Greek world, a coherent pattern of cultural activities surrounding such counters: a "counter culture". This culture is essential to the history of Greek numeracy.

Two universal human abilities combine in this culture. One, *opposition*, has to do with the hand, the other, *subitization*, with the eye and with visual perception.

Opposition is the most striking feature of the human hand, only very crudely approximated by a few other primates. Human fingers have a remarkable degree of freedom in their movements, including, to a certain extent, an ability to rotate the fingers towards each other. Furthermore — a feature most noticeable in comparison with other primates — the human thumb is not much shorter than the other fingers. Thus, humans possess the ability to bring the thumb directly against any of the other fingers. This is far from trivial. Try bringing your index finger against your middle finger, or against your palm; now imagine that this is the only kind of precision grasp you possess. Such an imaginary exercise makes apparent the centrality of opposition as a human skill. The tips of the fingers are extraordinarily sensitive, and they may be controlled with extreme precision. As a consequence, opposition allows humans to grasp and move precisely, with negligible effort, any object with an appropriate size, weight and texture.[11]

Opposition, as a purely morphological feature of the human skeleton, is meaningless: it is given meaning by a visual-tactile-kinesthetic cluster of perception and cognition, which we need not go into now. At any rate, while opposition has a purely morphological component, subitization is strictly cognitive. Current research suggests that it is no less universal: a feature of the human brain, analogous to opposition as a feature of the human hand.

Subitization is the ability immediately to perceive small clusters of proximate, similar objects, as clusters. In particular, the *number* of the objects in the cluster is immediately perceived, *without counting*. The size of clusters allowing subitization varies with the overall level of cognitive performance, and with other factors such as the special shapes of the clusters. Generally speaking, subitization breaks down at around four to five objects; with more than that, the number of objects cannot be known without some explicit mediation, e.g. counting.[12]

Opposition and subitization are universal features, shared by all humans. Some cultures make the step, obvious in hindsight, of combining these two features in a single practice, where small objects are moved to form clusters, related to numerical values. The culture surrounding such practice is what I call "counter culture". I now move on to discuss some of its manifestations in the Greek world.

2. COUNTER CULTURE

2.1. *Counter Culture and Greek Calculation*

Numerical practices are present in many settings, usually where something is done not for the sake of numbers, but for the sake of, let's say, economic exchange. Occasionally, practitioners to some extent isolate numbers from their practical setting, and pretend to treat them separately: whenever I count how many notes I am to hand at the cash counter, I ignore for the moment the vegetables in my carriage, and think just about numerical relations. I engage, in other words, in arithmetic and calculation, where numbers are manipulated and recorded as such (and not as numbers *of something else*). This is not the most common numerical practice, but it is obviously a basic one. For the connection between counters and numeracy, therefore, we may take as our starting point the connection between counters and calculation. This ancient connection is beyond doubt (it is not irrelevant to note that our word 'calculation' derives from the Latin 'calculus', which may be suitably translated in context by 'counter'). Yet the evidence — for reasons which will become obvious below — is not plentiful. Perhaps because of this relative lack of evidence, modern literature, too, has been scanty. The best survey in the English language seems to remain J. M. Pullan, *The history of the abacus*,[13] a slim volume by a gifted amateur. A much more extensive and up to date survey, though once again explicitly amateurish in character, is recently available in French: A. Schärlig, *Compter avec des cailloux*.[14] (This, the work of a mathematician, is especially praiseworthy for the analysis of the arithmetical structure of operations upon counters.) The only Greek scholar to have researched the subject is Lang, in a series of publications of fundamental value, published in the journal *Hesperia* between 1957 and 1968, to which I shall refer below. T. L. Heath, *A history of Greek mathematics*,[15] is still of some value. At any rate, while many questions on this issue are still open, there is no doubt that, in the ancient Mediterranean, calculations were frequently made by moving counters on a surface known as the 'abacus'.[16] We therefore need to look at the ancient, or western abacus.[17] In M. L. Lang's original publication in the field, "Herodotus and the abacus",[18] 14 abaci were listed from the classical Aegean world. In a later publication, "Abaci from the Athenian Agora",[19] the same author added two more from the Athenian Agora. A. Schärlig[20] extends the list to 30 objects, with largely the same pattern of distribution: nearly all from the Aegean world, most from Attica. (The furthest afield seems to be SEG XXIII 620, a third-century B.C. abacus found in Cyprus.[21])

In her original publication from 1957, as well as two later articles,[22] Lang went on to argue that some arithmetic features in calculations preserved in the literary tradition of classical texts may be accounted for by assuming operations on the abacus. Finally, while no ancient source discusses the abacus as such, there are many passing references that take it for granted.[23] Based on this archeological and literary evidence, a coherent picture of the physical shape of the ancient, western abacus and its usage may be suggested.[24]

The western abacus differs considerably from its better known, eastern counterpart.

The eastern abacus (still in use sometimes from Russia eastwards, and widely available as a Chinatown souvenir) is a framed arrangement of wires, along which beads may be moved horizontally.[25] It is a complicated instrument, whose manufacture requires considerable skill. The western abacus is technologically trivial. It consists simply of a flat surface on which lines are somehow marked, and of counters, of whatever kind, which may be placed and moved along those lines. It may perhaps help the reader to visualize the western abacus, if we note that backgammon offers a very close analogue. Backgammon, too, consists of (a) a surface, marked with lines, and (b) counters placed on those lines and moved from line to line according to given rules (we shall pursue this similarity further in Section 2.4 below).

In a handful of extant specimens, the lines are labelled by numerical values (typically 1, 5, 10, 50, ...), but otherwise such numerical values are left for the calculator to assign on an *ad hoc* basis, depending on the operation required. The rules of movement refer to these lines, and are extremely simple. This is indeed the essential difference between the eastern and western abacus. On the eastern abacus, no motion is possible *between* wires, the only operations being of the horizontal movements of beads *inside* wires. This makes the operation of the eastern abacus much more abstract and sophisticated. On the western abacus, movement is *between* lines, based on the definitional equivalences between numbers. Five times ten is fifty, and therefore five counters on the 'ten' line are equivalent to a single counter on the 'fifty' line; further, twice fifty is a hundred, and therefore two counters on the 'fifty' line are equivalent to a single counter on the 'hundred' line. Let us say, then, you start with four counters on the 'ten' line and a single counter on the 'fifty' line, and that you wish to *add ten*. You add a single counter to the 'ten' line, and have now five counters there; the rules allow you now to remove those five, and to exchange them for a single counter on the 'fifty' line. Now you have two counters on the 'fifty' line; the rules allow you now to remove them, and to exchange them for a single counter on the 'hundred' line. Here you stop, since no rules allow you to remove counters any longer, and so the calculation is complete: 90 + 10 = 100. This is essentially all there is to it. Subtraction and multiplication are somewhat more complicated, division much more so (the same is true with Arabic numerals, pen-and-paper algorithms): Lang claims to detect the impact of abacus operations on some ancient false divisions. On the basis of some archeological evidence, Lang goes on to suggest a final improvement: the abacus may have been used to hold more than a single number at a time. Thus, for instance, you would place two multiplicands on two different areas of the abacus, and operate on them while you construct the result of the multiplication on yet a third area. This may account for the relatively large size of some abaci (the largest, IG II^2 2777, is 1.49m × 0.754m).

Like Arabic numerals (and their Babylonian antecedents) the abacus is essentially positional: hence follows a certain abstraction. Just as it makes no difference, for pen-and-pencil operations, which absolute value the positions have (to add 1.345 and 1.678 is the same as to add 1345 and 1678), so it makes no difference, for the abacus, whether we move from 'fives' to 'tens' or from 'fifties' to 'hundreds'. If only for this reason, it makes clear sense to avoid marking the lines. It is true that the abacus is not as totally homogenous as are the positions of Arabic numerals: one must distinguish odd, 10^n lines, from even, 5×10^n positions. But such an alternate marking may easily be inserted on an *ad hoc* basis. We thus find that the western abacus has very little substance: really, no more than a row of scratches. The abaci listed by Lang were identified because, if not on the lines themselves, they had numbers marked at some other position of the abacus (perhaps to keep records during the operation). In the Greek world (unlike the Roman case[26]) no counters were ever identified as "abacus counters", and there is no reason to suppose any existed. Ordinary pebbles would do and, as we shall note below, the Greek world had a profusion of other counters of all kinds, all useable on the abacus. Further, while the extant abaci (with a few exceptions, e.g. two abaci scratched hastily on roof-tiles) tend to be made with marble, in ordinary circumstances a mobile board would have been more useful. Most probably, abaci were mostly made with wood, but this is pure guesswork, as naturally none survives. Ultimately, indeed, the very notion of the abacus as a clearly defined artifact is misleading. While scratches are useful, the lines can very well be *imagined*, perhaps referring to whatever irregularity the surface at hand may have. Thus any surface will do. The abacus is not an artifact: it is a state of mind. The western abacus was wherever there were sufficiently flat surfaces — as well as sufficiently many objects that the thumb and fingers could grasp. Probably more designated abaci can be found if we look for them with more attention. But perhaps designated abaci are less important than the skills that make them so easy to construct and use on an *ad hoc* basis.

Those skills are, as suggested in Section 1.2 above, primarily *opposition* and *subitization*. The role of opposition is obvious, but one should note also the central role of subitization. In all cultures where the western abacus was used, the numerical lexicon was decimal; and yet, it seems that the divisions of the abacus were always taken to represent a staggered sequence of 'fives and tens': 10^n, 5×10^n, 10^{n+1}, $5 \times 10^{n+1}$, ... This is obvious with the Roman numeral system, which is simply the easiest way to record the result of an abacus operation (each counter on the board is represented by the symbol of its assigned line: LXXXX + X = C).[27] The advantage of this staggered sequence is obvious once subitization is taken into account. The calculator never needs to estimate clusters larger than five. With some training, and especially given the control the calculator has over the shapes of the clusters, it is clear that the threshold of subitization can be pushed to beyond five so that the calculator never needs to count his counters explicitly. One can say that the operations of the western abacus are nothing but concrete subitization: clusters of counters are seen as unities — and are replaced by unities. The abacus is effective because it breaks calculation down into a sequence of operations, mental (subitization) and bodily (opposition) that, each, require no effort at all.

For the abacus to be effective, it had to be embedded inside a richer structure of cognitive tools and skills. We shall set aside the requirement for an arithmetical lexicon: this seems to be universal.[28] Nor need we consider here the emergence of decimal numerical lexica, since these too antedate the abacus.[29] In many occasions, however, it will be useful to make a more permanent record of the results of calculation, and this already calls for a special tool — numerical symbols. As suggested already, the Roman numerals are best understood as emerging from the abacus practice, and the same is true for the numerals used almost without exception in classical Greece.[30] These are known as "acrophonic", since they are based on the initial sounds of the numbers involved: Π for *pente*, 'five', Δ for *deka*, 'ten', etc. This system is precisely the same as the more familiar Roman system, of course without the modern "negative position" rule. Until the fourth century B.C., most extant numerals follow this system. From the late fourth century onwards, more and more documents use another, "alphabetic" system. This was sometimes used, in the Ionian world, in the period 575–475 B.C., but now it becomes nearly universal. In this alphabetic system, the Greek alphabet is arranged in three sequences of nine characters each (which calls for some special characters). The first sequence, from alpha onwards, stands for the unit numbers 1, ..., 9. The second sequence, from iota onwards, stands for the tens 10, ..., 90. The third sequence, from rho onwards, stands for the hundreds 100, ..., 900. For higher numbers, all sort of superscripts and special symbols may be used to turn those ordinary numerals into 'thousands' equivalents. It may be significant that the recorded abaci tend to come from the classical Aegean, and especially from Attica, the time and place of acrophonic numerals. However, I doubt if the *e silentio* has any meaning in this context, especially since the distribution of abaci largely follows the distribution of well-documented excavations.

At any rate, it should be clear that there is no evidence for any new calculation devices based directly on the properties of alphabetic numerals. Alphabetic numerals may be seen as a redundant version of Arabic numerals, where one needs to memorize different symbols for the tens and the hundreds. This blocks the use of paper-and-pen algorithms, yet certainly one could in principle arrange alphabetic numerals in ordered columns, allowing perhaps a certain help for mental calculation. None of this was noticed in the vast numerical records, extant on papyri.[31] Alphabetic numerals have some obvious advantages as records: as with Arabic numerals, numbers written with alphabetic numerals are shorter on average than those written with acrophonic numerals. Alphabetic numerals are even better than Arabic numerals in one feature, very useful in records: they allow an immediate recognition of the order of magnitude recorded. Perhaps, however, the main advantage of alphabetic numerals over their acrophonic counterparts is this: alphabetic numerals, but not acrophonic ones, may be read off as a record of the verbal, number *phrase*. Λ may be read as a special abbreviating symbol for 'thirty', a word in Greek as in English; B may be read as a special abbreviating symbol for 'two', a word as well; and ΛB may be read as a special abbreviating symbol for 'thirty two', a real phrase in the language. (In the Greek number phrase — unlike the English — the order would be two-and-thirty: this however does not change the basic readability of the alphabetic symbol as a number phrase.) ΔΔΔII,[32] on the other hand (say, "ten ten ten click click"), does not exist in spoken language. In Greek writing in general we see an important force at work: the tendency to make writing a record of verbal expression (this after all is the impetus driving the elaboration of the Greek alphabet). This force ultimately influenced the record of numbers, as well: this may be all there is to the rise of alphabetic numerals. At any rate, I do not believe that the disappearance of acrophonic numerals shows anything about the use of the abacus. Indeed, while acrophonic numerals are extremely simple to use with the abacus, there is no essential difficulty in using alphabetical numerals instead. (After all, Arabic numerals are easily used in modern demonstrations of the abacus.) On the arithmetical level, all those systems are decimal and essentially equivalent.

The analytical distinction between number manipulation and number record is useful. We may conclude this subsection by noting the following. Counters — and not written or spoken symbols — were the medium for the manipulation of numbers in the Greek world. They of course cannot function as a medium for permanent record, but their centrality as a medium for manipulation was such that, in the classical Greek world (as in the Roman world, well into early modern times) their use in the abacus shaped the form of the medium of record itself. This must be stressed: counters were not some *aid* to the manipulation of number, itself understood primarily in other terms. They were the medium of numerical manipulation *par excellence*, in exactly the same way in which, for us, Arabic numerals are the numerical medium *par excellence*. We imagine numbers as an entity seen on the page; the Greeks imagined them as an entity grasped between the thumb and the finger.

2.2. *Counter Culture and the Economy*

The essential feature of the Greek economy was its reliance upon counters. This is true in two ways: first, the reliance upon counters sets the Greek economy apart from any other independent ancient economy; second, the reliance upon counters permeated the Greek economy and shaped it in a deep way. I refer, of course, to a special kind of counters: small metallic disks, struck with symbols. Coins — for that is all coins are — were always appreciated as central to Greek history; the perspective of the history of numeracy may shed some further light on their significance.

In truth — like everything else concerned with money — coins are a mystery, and it is difficult to define their precise significance. J. M. Keynes, for instance, doubted their very importance. His remarks at the beginning of *A treatise on money*, may serve as an entry point to this subject:

> Money of account, namely that in which debts and prices ... are *expressed*, is the primary concept of a theory of money.... Such debts and price lists, whether they are recorded by word of mouth or by book entry on baked bricks or paper documents, can only be expressed in terms of a money of account. [Keynes notices the role of *State money*, and then goes on to distinguish *commodity money*, "actual units of a particular freely obtainable, non-monopolized commodity" and *representative money*, "something the intrinsic value of the material substance of which is divorced from its monetary face value".]

> The beginning of money proper is often associated by historians with the first coinage ... but I do not think the act of coinage effected so significant a change as is commonly attributed to it. It was, perhaps, the first step towards representative money.... But it is probable that the fundamental transition, namely the transition to chartalist or State money, long preceded it; just as the next important step, namely to representative money, was long subsequent.[33]

To paraphrase: the economic significance of money is in its serving as unit of account — an abstract, symbolic unit, which allows transactions to transcend concrete, face to face exchange. One major step in the evolution of money is its becoming State money — the rise of political guarantees for the unit of account (this happened early on in all complex societies). The second major step was money's becoming purely symbolic and representative (this is a very recent development in some modern states). Keynes's conclusion — that coinage does not constitute a major transition — seems, therefore, inevitable: coins are simply a way of packaging the commodity-money of precious metals. They do not involve any abstraction. At the abstract, symbolic level, coins are parasitic upon an antecedent symbolic domain, that of weights. Metal ingots were weighed throughout the ancient Near East, i.e. they were symbolically made equivalent to a certain number of units — the real unit of account. All coins do is to equate individual ingots with such symbolic units.

It is however absolutely clear, from a historical perspective, that Keynes was wrong. Coin cultures, from the Greeks onwards, were truly different from other commodity-money cultures. Why was Keynes wrong? Because, I suggest, his understanding of symbolism is anachronistic. His reference to records "by word of mouth or by book entry" is telling: Keynes thought of information as verbal, so that the numerical world of information concerning monetary exchanges was, for him, embedded within a world of verbal symbolism. Thus coins were merely a concrete way of instantiating written or spoken symbols such as, say, 'drachma' or 'pound'. This is a correct view of the role of coins in the twentieth century (the only century Keynes was much interested in). Numerical operations, in Keynes's world, are essentially mediated by relation to written symbols. *But in the Greek world the medium for numerical operations* par excellence *was counters*. Metallic counters, therefore, were not some concrete instantiations for a symbolic system: they were the symbolic system itself. (In exactly the same way, Arabic numerals in today's business papers are, directly, the symbolic system within which the numerical operations of the economy are expressed.)

Of course, Keynes's statements remain true: State money was invented before coins, and representative money was invented after them. What coins did however was extremely important. They brought money under one's thumb and, in this way — given the nature of Greek numeracy — they made money participate directly in numerical operations. Put quite simply, one would now count one's money.[34]

It would be tedious to show here that Greeks put coins together so as to calculate, on the spot, the sums of money they had paid or received. That coins can very easily be used in this way is the reason why they are still the dominant means of small-scale transactions, where ease of calculation is most important. It is necessary to say something, however, on their value as counters.[35] First — then as now — note that coins function on the basis of clustering and (to some extent) subitization. This is because the values of coins are nearly always some unit fraction of the larger unit to which they refer. In Greek practice, ordinary coins were at the scale between drachmas (and their small multiples) and obols (and their fractions), the obol being one-sixth a drachma. There were also one-and-half-obol, two-obol and three-obol coins, i.e. fourth, third and half drachmas; as well as half-obol and quarter-obol. One notes in general, in all areas of Greek numeracy, the absence of all fractions but the unit-fractions, and the use of unit fractions in Greek coin denominations should therefore come as no surprise.[36] The same principle is true of modern coins, and the American system, for instance, has a hundredth, a twentieth, a tenth and a quarter. Typically, however, only the quarter is perceived as such. In general, exchange is now based on our shared training in complicated decimal mental calculations, and coins are typically added together, not as fractions of dollars but as multiples of cents. The Greek denominations defy such mental calculations, and are clearly designed to be added together as clusters: three third-drachmas as a drachma, four quarter-obols as an obol, etc. The obol and the drachma serve as two steps, set between the smallest units one would use in an ordinary daily transaction (obol fractions) and the largest (several drachmas): very much like lines on the abacus. We thus see how, in ordinary, daily transactions, coins belong to the numerical world described in the preceding subsection. What else? That was the numerical world the Greeks knew.

The system is based on the manipulation of metals as counters, and this again calls for some comment. Precious metals are, in fact, slightly inconvenient as money: their very heaviness and smoothness — which make them attractive in the first place — also make them somewhat difficult to grasp and to manipulate. This is corrected by the coin. First of all, the coin — contrary to all hoarding instincts — breaks ingots into very small blobs of metal. These would be very easy to lift because of their lightness, but now the small size makes them difficult to manipulate. Thus the blobs are moulded into thin disk shapes, so that their diameter grows correspondingly, until, at around 10–20mm diameter / 1–2mm thickness, one obtains a typical Greek coin, say a silver drachma or didrachma, with roughly 10–20 grams weight.[37] Besides the thin disk shape itself (which incidentally, makes even the fingernail useful in lifting and manipulating the coin) the striking of the coin creates an irregular surface. This compensates for the smoothness of the coin — besides of course making it immediately recognizable as a counter of a given numerical value.

There is thus no surprise that coins quickly became the agents for the numerical exchanges involved in trade. Introduced sometime before 600 B.C., they became, already in the fifth century, the predominant medium of exchange, throughout the Greek Mediterranean, and across the entire spectrum of economic activity.[38] This very success, in fact, led to pressures resulting from the 'real' values of precious metals. No matter how flat you made them, silver coins in the smallest denominations required by ordinary, daily trade, could not be made wide enough to be easily manipulated.

Yet ordinary, daily trade was thoroughly penetrated by the coin economy. Hence the pressure, which may be suggested, e.g., by a well known joke in Aristophanes that assumes that tiny coins are ordinarily kept in the mouth (!)[39] — presumably, they are so tiny that one can hardly make sure otherwise they do not disappear. A glance at fifth-century Athenian obols and their fractions — specks of silver, no more — confirms the thrust of the joke: they have moved below the grasp of the thumb.[40] The ancient owner would have found it difficult to trace such coins — just as the moderns did, so that only very recently archeologists came to realize how widespread small-denomination silver coins were already in Archaic Greece.[41] Mainly in response to such pressures, then, *bronze* coins were struck, especially from the fourth century B.C. onwards.[42] Such coins are confined to the cities in which they are produced, because their value is strictly a matter of local convention. These are counters pure and simple, divorced from any reference to their commodity value as metal. As such, they approach in character the tokens found in great numbers, e.g. in the Athenian Agora. Made of bronze, lead or clay, the significance of those counters is difficult to ascertain; they seem to correspond to our vouchers or tickets.[43] Thus, for instance, it is a reasonable guess that some such tokens were handed out in advance of grain distributions: you would surrender your token in exchange for the grain distributed. In general, whether with silver, bronze, lead or clay, whenever exchange took place in the city it was measured by counters. (The case of tokens, however, already takes us beyond purely economic exchange to the domain of the political, of relations between citizens: we shall discuss this in the following subsection.)

It is necessary to widen our scope, from the lower ends of daily transaction to the higher ends of large-scale financial operations. These took place in the Greek bank — the *trapeza*, 'table' — no more than a glorified abacus. The operations of the banker required the table itself, a scale, a touchstone, the abacus and wax tablets or papyri for records.[44] Thus the banker would confront coins as either numerical concepts (hence the abacus) or as metals (hence the scale and the touchstone). First, for the abacus, one should note a complication — actually a rather minor one. As was already seen for obols and drachmas (and as is largely true for the higher denominations, minas and talants), the units involved do not fall into a simple decimal pattern. While the number ten has some significance, the number five has no significance at all, and multiples of six are especially common. The reason for this complicated pattern lies outside Greek history: as noted above, coin denominations are parasitic upon earlier, weight systems, which go back to the Ancient Near East. For obvious reasons, such metrological systems are extraordinarily conservative, and even today it takes enormous efforts by governments to effect conversions into decimal systems. Thus, all Ancient Mediterranean metrological systems ultimately derived from Mesopotamian temples, whose arithmetical culture was perhaps the most sophisticated the world has ever known. The peoples of the Mediterranean had to cope somehow with a numerical system designed by highly trained scribes, masters of sexagesimal operations.[45] This of course would make calculations somewhat difficult, but coin and weight calculations were effected by exactly the same methods as purely arithmetical calculations. Perhaps, in fact, this is why the abacus tended to be unmarked. An unmarked series of lines could serve equally well to represent 'fives', 'tens', 'fifties', etc., or, say, 'obols', 'drachmas', 'ten-drachmas', 'minas', etc. Several literary references to the abacus envisage just that, while some of the numerical markings on the edges of abaci belong to this family of symbols.[46] All one needed to do was to adjust, mentally, to the correct equivalences between neighbouring lines — and one had enormous experience with such equivalences, in daily economic life.[47]

Now to the touchstone and the scales. In large-scale transactions (as well as in any money changing), it would become necessary to return to think of the coin as a commodity — as metal to be assayed (by the touchstone) and weighed. At first glance, the metal as such might be considered to provide a rock bottom for this symbolic system, going outside the abstract world of counters and into the real world of absolute weights. One might think that absolute weights are the external grounding for the hermeneutic cycle of coins exchanged for coins, of counters exchanged for counters. This first impression is wrong: there is no way out of counters. To operate the scales themselves, the banker would simply use *a further set of counters* — weights — which are the only "external grounding" for coins: one set of counters, coins, is weighed by being made equivalent to another set of counters, weights.[48] Then the counters of weights are counted and arithmetically manipulated inside a counter based calculation, always within the hermeneutics of counters: *il n'y a pas de hors-jetons*.

The general parallelism between coins and weights must be stressed: they are the two sides of a single equation. Just as coins are the currency of demand, units for expressing the purchasing power of the buyer, weights are the currency of supply, units for expressing the goods offered by the seller. It is for this reason that the ancient metrology is so complicated: because new weight standards could have been issued from time to time, essentially as inflationary measures.[49] This is important to realize, precisely because we tend to think of weights and measures — probably under the influence of the scientific use of standards — as external, 'real' units. At least in the ancient economic context, they are better understood as part of the symbolic numerical system employed by a culture: this was most transparent for Greek practitioners, taking for granted the embodiment of weights by counters.

One should distinguish of course between kinds of weights. In many daily transactions, reference is made to relatively large weights, which cannot be reduced to easily manipulated objects. Kilograms — heavy stone weights, requiring the two hands to lift them — were used in such cases.[50] But it is significant that at the hundred-grams range (and below) the Greeks usually had recourse to lead, using its high specific gravity to produce graspable counters (bronze could be used as well, especially with small weights). The cylinders could be made thicker and thicker, now making sure the diameter would not become *too large*: generally speaking, the diameters of lead weights were kept at no more than six centimetres, well within grasp. Once again, the values of weights were given as unit fractions of the basic weights, and we are thus fully within the system elaborated above for coins.[51]

We have therefore seen several ways in which numerical operations in the economic sphere are mediated by counters. It only remains to note that no other media for operations are to be found. Just as there is no trace of anything resembling our pen-and-paper algorithm in the realm of pure calculation, so there are no specific written techniques used in the economy. In particular, there are no special techniques of bookkeeping. The ancient accounts — of which there are many — are an unstructured sequence of inventories and records of transactions, which do not allow any operation to be conducted at the written level.[52] An ancient account was a mnemonic and legal aid, for storing information in a stable form; to operate with this information one had to extract the information from the account and to manipulate it in some other, non-written way. We are thus led back to the banker's table with its sets of counters. We have little indirect evidence: but it is a fact that the only evidence we have, for numerical operations in the economic sphere, involves the manipulation of counters. And, in fact, we have plenty of *direct* evidence for such manipulation: we have ancient coins.

2.3. *Counter Culture and the Political Domain*

The preceding subsection might be misleading. I have chosen to concentrate on a single layer of the ancient economy — the world of urban, monetary exchange. This world has become fully numeric; but below that there also persisted a rural world of subsistence farming, a world where the circulation of coins was limited and so, probably, was the influence of numeracy. This however does not diminish the interest of the history of numeracy for the economy: the interest is precisely in the differential presence of numeracy, and its possible significance. I shall not pursue this question here, but it may serve to introduce the issue of numeracy in the political domain. In the political domain, the differential presence of numeracy is the heart of the matter. With the economy, coins allow a simple generalization: numeracy made a huge impact on the Greek economy. In Greek politics, numeracy made a more nuanced impact, which is best understood in terms of the pattern created by the decisions, to use numeracy in specified contexts (and not others), in specified formats (and not others).

Let us first mention the ways in which numeracy may be present in the political domain. (1) Numeracy may be involved in the *constitution*: the political structures may be defined in numerical terms (the constitution is usefully subdivided further in two: the definition of (1.1) the *citizen body*; and the definition of (1.2) *political institutions*). (2) Numeracy may be involved in the *decision making process*: decisions might be made through numerical acts. (3) Finally, numeracy may be involved in the *political content*: the subject of political debate and action may be numerical. To make this more concrete, I briefly survey some examples from the Athenian democracy:

1. The citizen body in Athens was formed, from very early times, in orders defined by census of income. The citizen body was further defined (much more important in the mature democracy) by age classes. Both formations made reference to numerical terms ("500 bushels", "age 18"), and it is an important historical question, how numeracy

was involved in applying those divisions in practice. Further, the fundamental structure of the citizen body was the Cleisthenic division into tribes, ridings and demes. At the level of tribes (but not at the possibly more important level of demes) the number chosen for the new division was decimal, namely ten: this was a successful decimal revolution (M. H. Hansen's comparison to the French Revolution in *The Athenian democracy in the Age of Demosthenes* is telling in this respect, too).[53]

2. Athenian magistracies and law-courts were, almost without exception, collegiate bodies where the number of members was, at least in theory, explicitly defined. Almost always, the numbers were multiples of powers of ten, or simple modifications of them (in the law-courts and other crucial magistracies the rule was $10n \pm 1$: an odd number, assuming everyone votes, would rule out the possibility of tied votes).[54] Such numbers frequently made direct use of the decimal structure of the citizen body arranged in tribes (e.g. the 500-member council was formed of 50 from each tribe). The constitution also made reference to a calendar whose months, in the political context, were made decimal (not even the French Revolution had accomplished *that*). Thus the presidency of the annually chosen council would rotate between the ten tribal groups, each group holding the reins for a decimal 'month' (of 36 or 35 days). The entire political calendar (regular and special votes, etc.) was fixed in terms of such decimal months.

3. A principle of democracy is decision by majority; arguably, its essence. This is ultimately a numerical concept, though its implementation in practice may involve varying degrees of numeracy.[55] Further, a special kind of decision is choice of magistrates: this was in some important contexts done, in the Athenian democracy, through majority votes, but more often this was done through election by lot. Election by lot, too, may be seen as a numerical operation (more on this below). The complicated processes of votes and lots were very visible in the Athenian democracy, and they offer some interesting examples of counter culture: I shall concentrate on them below.

The content of political decision making was — in Antiquity as ever — dominated by the allocation of resources, on the one hand, and foreign relations, on the other hand. The allocation of resources belongs to the economy and reflects its numeracy. Further, in the Greek *polis*, perhaps the one most important part of political life was the interface between the internal allocation of resources and foreign relations. Above all else, Greek citizens debated the profitability of waging wars: perhaps the emblematic decision of the Athenian democracy was to follow Themistocles's advice and use the Laurion windfall to fit a navy. Agamemnon did not need to budget the siege of Troy: Greek political life proper begins with the quantification of warfare. In general, the role of numeracy in ancient warfare deserves a separate study (both Plato[56] and Polybius[57] mention numeracy's practical value in a predominantly military context), and I leave this aside here.

This brief survey may serve to indicate the need for a history of numeracy in the Greek political life. I now move on to note some of the ways in which this numeracy may have been shaped by counter culture. I concentrate on a single aspect of the

Athenian democracy, namely its decision-making procedure.

It should be stressed straight away that group decision making might be characterized by varying degrees of numeracy. Even where majority decision is in some sense the ideal, this ideal may be implemented with more or less numeracy. The Spartan shouting vote (whatever its historicity) may serve as example of non-numerical decision making: the shouters are not measured as discrete, countable groups of people, but meld their voices into a single unit, estimated as a whole.[58] This then is a qualitative and not a quantitative vote. In democratic Athens, votes were in principle strictly quantitative, and voters appeared as a discrete group of individuals. That each citizen became just that — a unit to be counted, obliterating all qualitative differences — was a principle fully grasped by the enemies of democracy who noted its 'arithmetical' nature:[59] democracy, literally, is where everyone counts. But even then, there are different ways of counting, related to different uses of numeracy. The Athenian democracy knew two forms of vote counting: by show of hands, and by counters. Show of hands was used in all ordinary votes of the entire citizen body, with a few exceptions such as ostracism; counters were used elsewhere and especially in the law-courts. The word for the assembly's decree (i.e. the result of votes by the entire citizen body) was *psephisma*: this is derived from *psephos*, in context meaning 'counter', and many scholars assume that in early times even the assembly voted with counters. At any rate, counters were a central metaphor for political decision-making; but show of hands was essentially different from vote by counters. In a large assembly of some thousand voters, a strict count of hands is impossible. Instead, a special college of magistrates, nine *proedroi* (10 – 1: a tied vote had to be avoided) were responsible for *estimating* the outcome of the vote.[60] This show of hands, then, lies somewhere between a shouting vote and a strictly numerical vote.

On the other hand, strictly numerical votes always involved the use of counters. To stay with the entire citizen body, then, we may begin with its most sensitive vote, ostracism. This took its name from the type of counters it used — ostraka. The Athenian citizens would (once a year at most) take up ostraka — shreds of broken pottery — to inscribe the names of individuals they wished to see expelled from the city. These special counters were deposited as the vote proceeded. At the end of the vote, the contents of the urns were counted, first to ensure the quorum of 6000 voters, and then, only if the quorum was reached, to found who was exiled — the citizen whose name appeared most.[61] The ostraka counters were almost certainly counted on abaci using other counters, and the entire operation was not much different from that of a banker, receiving many coins of different kinds, and calculating their sums with his counters. Note however that since names had to be inscribed on the ostraka, they had to be larger than other, ordinary counters.[62] In this they are comparable to the other important type of inscribed counter used by the Athenian democracy, the *voter plate*. Citizens were issued small plates (first bronze, later wood), with their names inscribed:[63] these "identity cards" were used not so much to identify citizens, as to chose them by lot. Choice by lot was made by manipulating those special bronze or wood counters. Choice by lot should be mentioned at this point, since it was a central operation of the Athenian democracy, arguably as important as majority decision.

It is difficult to assign choice by lot to any cognitive domain, since its entire purpose is to lead to *arbitrary* events — events *un*influenced by human skill. In fact, much skill, essentially numerical and mathematical, is required to produce arbitrary events. One needs to construct mechanisms so that their structure mirrors the set from which selection is to be made; then the operation of selection needs to be designed so that no agent will be able to control the results. The Athenian 'kleroterion' — a complex mechanism known from detailed literary descriptions and from several archeological finds[64] — was an ingenious solution to this problem. Its principle was the parallel operation of two sets of counters. On the one hand, sets of citizen-plaques are arranged serially into slots (the structure of the kleroterion means that they were taken in groups of five). On the other hand balls or dice are arranged serially, inside a vertical tube: they are arranged *independently from the plaques*. As balls or dice are rolled out of the tube, one by one, their markings (e.g. white or black) are correlated with the corresponding set of citizen plates, with predetermined consequences. So, for instance, assume you need the operation to yield 100 persons from a larger group. Put inside the tube as many black balls you like, and twenty white balls; insert the citizen plates, independently, into slots arranged in groups of five; and now go, five by five. Whenever you hit a black ball, the plates are dismissed and their owners go home; whenever you hit a white ball, the plates are taken and their owners are selected. In sum, then, we see the numerical nature of the operation (e.g., it enshrines the role of the multiples of five in the Athenian democracy; it also requires some basic numerical skills in the arrangement of the numbers of counters). Naturally, this numerical operation is directly based on counter manipulation.

Ostraka and voter plates were important counters, but the most important one — the one that symbolized the Athenian democracy above all — was the *psephos* (whence, as noted above, the word for decree, *psephisma*). The basic meaning of the word is 'pebble', but the actual materials of psephoi varied. The one constant principle was their size: the small, thumb-and-finger size. With the single exception of ostraka, psephoi were used whenever the Athenian democracy had recourse to counted votes.[65] This happened only rarely in the assembly (e.g. in votes for granting citizenship), but such psephoi-based, counted votes were the hallmark of another institution crucial to the Athenian democracy, namely the law-courts. The Athenian law-courts were a kind of collegiate magistracy of *dikasts*, formed ad-hoc, through choice by lot, for each trial; they had relatively large, $100n + 1$ (usually in the range 201–501) memberships.[66] Their procedure was based on public debate between plaintiff and defendant, followed by the dikasts casting their votes one by one. The votes were then counted on an abacus; one wonders whether the voting psephoi themselves were used as counters.[67]

Boegehold has suggested that at the very first psephoi might have been cast at plain sight.[68] His evidence includes several vases with imaginary scenes of voting, where the voters approach a raised platform and deposit on it what appear like small stones (this is rather like a Jewish funeral). In historical times, the dikasts used various forms of secret ballot, e.g. by depositing their psephoi inside hidden urns (so one could not see where the hand tended), or by emptying both hands inside urns, only one hand actually holding a psephos. (The psephoi themselves seem to have been natural objects, either actual pebbles or seashells.[69]) None of this is truly satisfactory, and the Athenians used for some period around the end of the fifth century a written version of voting: wax tablets, inscribed by either long or short scratches, were deposited inside a *single* urn.[70] Interestingly, this solution was discontinued in the fourth century, when the final form of the Athenian ballot was reached. This is a true masterpiece of functional design, rethinking the counter into its essence: the grasp of the thumb and the finger. The design goes like this. Have wheel-like counters, their diameters a few centimetres, cast in metal (usually bronze, sometimes lead). The axle of the wheel is considerably elongated, almost as much as the diameter of the wheel. The counter is therefore grasped with the thumb on one end of the axle, a finger on the other end. This grasp is now made functional for ensuring the secret ballot. Some counters have their axles hollow, some have them full; but with the counter grasped, they are indistinguishable. Now give each dikast two counters, one 'hollow', one 'full'; designate the 'hollow' as vote for the plaintiff, the 'full' as vote for the defendant; finally designate two urns, one 'valid', the other 'invalid'. Thus, if you wish, say, to vote for the plaintiff, you approach the urns with your counters grasped and release them inside the urns: the 'hollow' in the 'valid' urn, the 'full' in the 'invalid' urn. *Both urns* would now be counted (in an echo of sort of double entry book-keeping); the counting would be done on specially designed tables, with pierced holes (where axles are inserted) so that, for once, we move away from the simple abacus to a more complicated counting instrument.[71] In short: the law-courts were run by the grasp of the fingers — and the amateur of the law-courts caricatured in Aristophanes's *Wasps* woke up with his fingers in grasping position....[72]

The typical day in the law-court would start, therefore, with the dikast handing and receiving back his voter plate (counter 1), according to the ball in the kleroterion (counter 2); then receiving and handing back (in the mature system) two voting ballots (counter 3). To these we should add another thread of counters running through the day: upon entrance, the dikast would be given a randomly chosen coin-like, bronze (or lead) token, which guided him to his seat (to prevent factions, seating had to be randomized) (counter 4). Upon voting, he would gain another token of a similar type (counter 5), to signal his fulfilling his civic duty; this token would finally be exchanged for the dikastic pay: originally a third-drachma coin, later a half-drachma (counter 6).[73] (Counters 4–5 belong to a wider set of counters used throughout the Athenian civic life, e.g. for entrance to the theatre: I have mentioned such counters briefly in the preceding subsection, since they frequently have an economic significance — compare counter 5. Often, like counter 4, they had a more procedural role.) To sum up, we may refer to Boegehold's summary of the four "bases of the Athenian popular court system", which were, according to him, "large judging panels, allotment of dikasts, pay for dikasts, and the secret ballot".[74] The last three were permeated by counter culture, all four were permeated by numeracy. (The judging panels were not simply *large*: they had specified numbers of members.)

This subsection merely touched on the surface of the role of numeracy and counter-culture in a single Greek political context. Some results however are clear. First, as suggested at the start of this subsection, one notices a pattern: numeracy was strongly involved in some activities, less so in others. Further, these activities were also marked by counter culture: numeracy entered the Athenian democracy to the extent that counter culture entered it. Show of hands, a non-counter activity, was also a non-counted activity. A comparison with contemporary political life might be suggestive in this respect. Numerical statements are central to contemporary political discourse, e.g. when politicians refer to the mandates they had won, or in speculation concerning possible outcomes of elections, coalitions, etc. Nothing of this is present in the more obviously 'political' aspect of the Athenian democracy: no strategos refers to the votes he won, no orator complains that, say, "the decision to ravage Mitylene was made with a mere 51% vote".[75] In the Athenian legal life, on the other hand, references to such numbers are relatively frequent.[76] The difference is obvious: the assembly, in general, simply did not *produce* numbers. Roughly put, we may say that the Athenians were non-numerical in the general affairs of the city, and numerical in important votes concerning individuals (that is, votes in the law-courts, as well as ostracism and grants of citizenship; the assembly voted on personal matters such as granting honours for individuals, but those are clearly of less importance). This may reflect a sense that important votes concerning individuals should be taken extremely seriously, but then again, many of the general affairs of the city were much more important. Numerical operations may have been important to block individual grievances (which would indeed have been raised most strongly in the numerical votes); more important, I suggest that counter culture had driven numeracy, in this case, rather than the other way around. In the mature democracy at least, counter votes were all at least potentially secret, and this, rather than their inherent numerical nature, may have constituted their immediate significance for the Athenians. An essential feature of the Athenian democracy was that the ordinary public — people of mostly modest means — made decisions concerning the élite people who were usually wealthy. Everywhere in the Athenian constitution, therefore, we see an obsession with the prevention of bribes. Hence, to a great extent, the use of counters in secret votes and in choice by lot. As it were: the bronze counters of the Athenian constitution were there to block the silver counters of the Athenian élite.

This may be one reason for the differential use of counters in the Athenian democracy. On a wider view, we must note a major consequence of the nature of Greek numeracy. The fact that numeracy was tied to concrete objects such as counters greatly contributed to the concrete, face-to-face nature of Athenian politics (which had, of course, other motives as well). Political bureaucracies are necessary, literally, to keep count. But the Athenian political procedure was based on individuals presenting concrete tokens of their votes. The pool of eligible dikasts, therefore, existed not so much as a centralized list, kept by the Athenian bureaucracy, but as a very large, decentralized set of counters — citizen plates — spread across Attica.

Finally, we should note that numeracy in the Athenian democracy must have been differential. An important historical question, which I shall not pursue here, is how differential was its social reach. In democracy, everyone counts, in the sense that everyone is being counted. But does everyone count in an active sense? Did every Athenian citizen have a grasp of the numerical mechanisms of the constitution? One suspects that to a large extent they did, if only because of the presence of numeracy in daily urban exchange. At any rate, an obvious comparison with democratic Athens would be Sparta, and there are two main contrasts between the two: Sparta's lack of truly democratic institutions; and Sparta's lack of coins.[77] Ancient observers would also note the low level of numeracy in Sparta.[78] In this article, I suggest the thesis that these three are interrelated: Greek numeracy, Greek coinage and Greek popular politics, all belong to a single pattern. I leave this thesis here as a mere suggestion, and move on to describe the presence of counter culture in the symbolic domain.

2.4. *Counter Culture and the Symbolic Domain*

In a sense, I have throughout discussed the symbolic domain: I have described how counters served as vehicles of meaning, in the arithmetic, economic and political domains. It comes as no surprise, then, that the Greeks saw *meaning in counters*. Looking at counters, Greeks would perceive not pebbles, but signs; not mere matter, but meaning. The best place to see this is with divination. In every culture, a certain set of objects and events is singled out by being used for divination, that is by standing for something beyond itself. Divination is thus the place where a culture's sense of the semiotic — that which it takes to carry meanings — becomes visible. And it is therefore interesting to see the various sources of meaning in Greek divination.

It goes without saying that the dominant source of meaning in classical Greek cluture was speech — kept on papyrus rolls, or spoken. The prophet of Aristophanes's *Birds* reaches cloud-cuckoo-land armed with rolls of prophetic speech; and we typically think of the oracles of Dodona and Delphi as speech.[79] There was more to oracles: in many centres of divination, including Dodona and Delphi, use was made of divination by lot. The inquirer would put a question, and the prophets would offer a simple answer by picking a pebble (or a bean?), randomly, from a set of suitably marked counters: no subtle oracle, that. (This is essentially the method used in the Athenian choice by lot.) Choice of a counter among many is one way of obtaining a random result, i.e. a result apparently free of human agency and therefore divine. The same effect may be obtained with a single counter, if it is thrown at random and its fall recorded. Different shapes of solids allow different combinations: the 1-of-2 variety of flat discs obtained by tossing a coin, the 1-in-6 variety of modern dice. In the ancient world, 'astragali' — animal knucklebones of crude cubic shapes — were commonly used (incidentally, they represent a 1-of-4 variety, since they do not fall naturally on two of their sides). Astragali were frequently used for divination. It is worth quoting Pausanias's description of one such oracle:[80] "[They practise] a mode of divination by means of dice and tablet. The person who inquires of the god prays before the image, and after praying he takes four dice, and throws them on the table.

There are plenty of dice lying beside the image. Each die has a certain figure marked on it, and the meaning of each figure is explained on the tablet." One notices the double artifact, containing both designated counters and designated surface. This duality — strongly reminiscent of the abacus — is repeated elsewhere: counters were placed on special, holy tables, in minor cultic centres as in Dodona and Delphi themselves. It does seem however that, in the leading centres, speech overshadowed counters, at least in the official religious ideology: this is a useful hint for the place of counters in the larger system of Greek semiotics.[81]

Counters as a semiotic system could have been used in more explicit, reflective ways. One thinks in particular of so-called 'Pythagorean' authors with their use of figured numbers. We have several sources, from Aristotle to late Antiquity, where pebble-like patterns on surfaces are taken to represent specific numbers and numerical relations, always within the context of a philosophy where numbers have a special value. This phenomenon is much misunderstood. Scholars of ancient mathematics and philosophy used to believe once in the existence of an early Pythagorean "number atomism", where the world as a whole was composed of configurations of monads, understood as arithmetic patterns. This was supposed to have been shattered by an ancient crisis of foundations: the discovery of incommensurability (which implied not everything could be measured by a single unit, hence no arithmetization of the universe). Consequences followed for the history of mathematics (empirical arithmetic gives way to axiomatic geometry) and philosophy (early numerical Pythagoreanism gives way to geometrical Platonism).[82] Nothing could be further from the mark. Following the work of W. Burkert,[83] the notion of an early Pythagorean mathematics has become untenable. No crisis of foundations is noticeable in ancient Greek mathematics, where the discovery of incommensurability must have been a source of delight, not anguish (the counter-intuitive was treasured, not spurned).[84] All the evidence for figured numbers is later than the discovery of incommensurability, and it never comes from a context even remotely like Euclidean, theoretical arithmetic. Yet the phenomenon of figured numbers is very easy to understand in the context of counter culture. What we see is that whenever philosophers assign an important role to the numerical in their world-view, this comes together with reference to pebble-like patterns on surfaces. And what else should we expect? Pebble-like patterns on surfaces were the dominant ancient numerical symbolic system. It is natural for Nicomachus to draw enumerated, dotted figures: just as it is natural, say, for a Kabalist to play with the characters of the Hebrew alphabet. The significance of 'Pythagorean', figured numbers, is in offering another example of the relation between numeracy and counter culture, and for counters as a semiotic system.

Yet another evidence for counter culture — this time more from a cognitive more than from a semiotic perspective — may be seen in yet another symbolic activity, that of games. The history of games is of particular interest for cognitive history: games are played because people take pleasure in exercising those skills in which they are proficient. Thus the history of games indicates the historical development of skills.[85] In the Greek case, the most important games were sports such as running and wrestling, and clearly the skills the Greeks valued most were physical to the exclusion of any significant cognitive component.[86] (Incidentally, this is very different from modern spectator sports, which often rely upon complex numerical rules.) Still, more cognitive games are sufficiently attested; they belong to the pattern we have seen so far in this article. It seems that Greek games proper (as opposed to sports) were all board games, falling into one of two varieties: 'kubeia' and 'petteia' (the two words may have referred to specific games and not to kinds of games, but the generalization still holds). Kubeia-type games were race games, where counters were moved across a marked board. The race was governed by throws of dice ("cubes", hence the game's name). The motion of counters across a marked board is of course directly reminiscent of the abacus. To explain its operation, I have compared it to backgammon: as a matter of fact, backgammon is a direct descendent of the Greek kubeia[87] which in turn, I suggest, reflects in some sense the abacus. However, while the motion of backgammon is very reminiscent of the abacus, it is in an important way cognitively distinct from it. The beauty of the abacus lies in its precise rule-based operation, free of any arbitrariness, but kubeia (to introduce a ludic element of suspense and competition) use dice and thus introduce arbitrariness. Truer to the spirit of the abacus, then, is petteia ("game of small pebbles"), where the counters are moved according to strict rules. The ludic element consists in the players' choice from a range of optional moves available at each stage. Thus the game of fortune is transformed into a game of skill. The details of the ancient petteia are very unclear, but the game was probably some sort of antecedent for draughts, even chess.[88] A moment's reflection on those games — backgammon, draughts, chess — reveals their essence as games of figured numeracy (an essence appreciated by Plato[89]). Such games operate on the calculation of positions of counters on a discrete, numerically defined board; hence the relative ease with which they are analysed by computer programs. At a certain abstract level, the computer, the modern player and the ancient player, all display the same set of numerical skills. The computer, though, is blind to the thrill of the game — a visual and tactile thrill, shared between the eye's appreciation of the pattern on the board and the thumb's triumph of a captured piece.

On the other hand, the Greeks did not have other types of cognitive games. To conclude this survey, I notice one kind of game whose history may be suggestive. Perhaps the most popular type of game in early modern Europe was card games: games involving the manipulation and calculation of rectangular pieces of paper inscribed with various symbols, frequently numerical. The popularity of such games, it should be noted, well preceded the invention of print (which of course made them even easier to manufacture). They are first attested in Florence, in the late fourteenth century, and hand-made cards are well attested from the fifteenth century.[90] It is tempting to put such cards side by side with other developments such as the rise of systematic, coordinated book keeping, the appearance of banknotes, or the phenomenon of 'scuole d'abaco', literally "abacus schools", where pupils were taught explicit rules for written manipulation of numbers. Suddenly, Italy of the fifteenth century was flush with numerical symbols inscribed on paper. In more general terms, it has been suggested that a new 'calculation' mentality arose in the capitalist communes of Italy.[91] I shall return to mention such claims in Section 3 below. My suggestion in this article is mainly methodological: that such general claims should be studied, not as statements about numeracy as such — whether people do things with numbers or not — but as statements about the *cognitive history* of numeracy — the precise sets of skills and tools in whose context numeracy is implemented. I shall now try to unpack this methodological statement.

3. TOWARDS A HISTORY OF GREEK NUMERACY

As suggested by the preceding remarks, the history of numeracy has a very wide scope indeed. To gain a proper understanding of Greek numeracy it would be useful to compare it to other forms of numeracy, in other cultures. In the most direct way, it is important to put Greek numeracy in the context of its antecedents in the ancient Near East.

In excavations of Near Eastern sites from 8000 B.C. onwards, small clay objects were often encountered: spheres, cylinders, cones, double cones and tetraheders, of little obvious aesthetic value or practical function. They eluded interpretation until, relatively recently, it became possible to correlate them with later, inscribed documents. Schmandt-Besserat has led much of the study of these objects, and offered a bold thesis: that such counters are not simply correlated with some later inscribed documents, but are the direct antecedent of the whole of inscribed clay and writing in Mesopotamia.

I sum up the argument as presented by Schmandt-Besserat.[92] Counters were first used for concrete accounting (probably in the context of early city-formation, with its incipient forms of taxation). Individual counters would stand for individual counted items; different kinds of counters would stand for different kinds of commodities. The term 'token' is indeed most appropriate. Further, beginning in the early fourth millennium B.C., such token-accounts were sometimes made into more permanent archives by being strung together or by being enclosed in clay envelopes. These envelopes are sometimes inscribed with signs related to their contents, and this in turn may have suggested the next stage: inscribed tablets with signs referring to tokens. At around 3100 B.C., special symbols standing directly for numbers suddenly appear: instead of repeating the symbol for the token 'sheep' five times, one would inscribe the symbol for 'five', followed by 'sheep'. This belongs to the immediate context of the emergence of cuneiform in Mesopotamia and may be considered one of its key constituents: certainly, for a long time to come, cuneiform writing was almost wholly used for such accounts. Finally, note that tokens continue to be used in later times, within the same symbolic context. (In fact, the breakthrough in the study of ancient counters was the interpretation of a token-envelope accompanied by cuneiform, dating from the mid-second millennium B.C.[93])

Schmandt-Besserat's grand synthesis was sometimes criticized for relatively minor points, and indeed the simple stream of evolution sketched above must have corresponded, in reality, to a complex pattern of separate developments.[94]

It is certainly improbable — nor does Schmandt-Besserat imply this — that tokens of a given type had the same meaning, across the chronological and geographical range of her study, from 8000 to 3000 B.C. and from the Mediterranean to the Persian heights. What Schmandt-Besserat has deciphered is not a code, but a culture: the Ancient Near Eastern counter culture. There can now be no doubt that, throughout the area and period studied by Schmand-Besserat, small clay counters functioned as a symbolic medium; and that this medium served predominantly in a numerical, economic context.

Here then is another counter culture. What is more, the Near Eastern culture was the direct antecedent of Greek numeracy, as for so much else in the Greek world. The word 'abacus' may well be Semitic in origin. The weights and measures within which Greek numeracy operated were all Near Eastern; coins were first struck in Lydia, on the border of the 'Greek' and the 'Oriental', whatever the two terms may mean. It is worth noting that both kubeia and petteia are Ancient Near Eastern games in origin: backgammon and chess begin their evolution in Mesopotamia and Egypt, not in Greece.[95] And generally speaking, cognitive tools and media are conservative — and easily exported. They do not tend to disappear, but to accumulate. Oppenheim discovered a token-envelope accompanied by cuneiform writing: just as we go on playing chess and backgammon even though we have accumulated along the way card games, not to mention video games, with each kind of game representing a separate layer of cognitive history. Cognitive tools and media operate in the first place just because they assemble basic, universal human capacities. There is no special reason why they should be confined to limited times or locations.

Why study the *Greek* counter culture, then? Why even see this as an *historical* study? Given that cognition is in some sense universal, cognitive history seems impossible. All that can be told is the repetitive story of the unfolding of the same human capacities, displayed time and again. But this impression is of course misleading — as we have learned from the history of literacy. Indeed, cognition may be, in a significant way, universal, and therefore cognitive tools may be easily transferred between cultures. But they are historical, in two ways: first, they have to be invented, and exported, in the first place; second, and much less obvious, they function in a given culture *within a context of other cognitive tools.*

To conclude this article, I shall try to explain this thesis, of the *contextuality of cognitive tools* — that cognitive tools may have different meanings depending on the wider system of tools within which they are implemented. It is this, above all, that makes cognitive history necessary. For if the thesis of the contextuality of cognitive tools is true, than cognitive tools — and therefore cognition — simply do not exist except in history: in the historically given context of their use.[96] To introduce this thesis, I now tentatively offer a very brief and schematic map of the historical contexts of numeracy (needless to say, a more responsible description of such a vast field cannot be attempted in such a brief space, and my purpose is merely to illustrate an historical approach).

To allow for easy mapping I use a simple two-by-two grid:

	Numerical	Verbal
Manipulation	Calculation	Argument
Record	Account	Document

I distinguish two areas — numerical and verbal — and two approaches — manipulation and record. Each culture has some form of calculation, argument, account and document (often, documentation is purely verbal and is based on witnesses: this is a form of documentation, based on such basic tools as memory and social relations). Also, in each culture, different media may be used for each of the four types; often, there would be social differentiation based on accessibility to such media.

In the Neolithic Near East, clay counters often served as a medium for accounts, while documents and arguments were oral and so (possibly) was calculation. With cuneiform writing, a new medium — having an extremely narrow social accessibility — covers both accounts and documents. Many Near Eastern civilizations are characterized by these two divides: between written record and oral manipulation; between the few scribes (and their patrons) and the many illiterates. In the classical Greek world alphabetic writing becomes a major tool, with a relatively wider access. It now covers not only accounts and documents but also, to a growing extent, argument. (Acts of verbal persuasion, presented in writing, are the essential feature of Greek science and philosophy.) What it does not cover is numerical manipulation or calculation. Hence the hiatuses that characterize Greek numeracy: between calculation and accounts, on the one hand; between calculation and argument, on the other hand. Those hiatuses will be bridged, in incipient forms, only in Medieval Chinese and Muslim cultures and, finally, in early modern Europe where, ultimately, the written page is a medium serving calculation and argument, account and document.

I shall not try to argue the consequences of the Greek hiatuses (in particular, not being a specialist in ancient economic history, I cannot claim to make such an argument for the hiatus between calculation and account). But to suggest the kinds of issues for which the history of numeracy might be relevant, at least two possible consequences might be mentioned. First, the Greek hiatus between calculation and account may have shaped to some extent both Greek economy and Greek politics. The absence of double-entry book keeping in the Greek world may not have been as historically significant as originally claimed by de St. Croix in 1956.[97] But there does seem to be a qualitative difference between ancient and modern capitalisms, and this is probably related, among other things, to the new, systematic use of accounts *as tools for calculation*, in modern capitalism. In the political domain, I have suggested above that the concrete nature of Greek voting may have contributed to the overall concrete, face-to-face nature of the ancient democracy, with its relative lack of central bureaucracy. To repeat: the above is no more than a brief suggestion made by a non-specialist, only so as to illustrate the methodological claim that the history of numeracy might, perhaps, be of consequence to economic and political history.

Second, the hiatus between calculation and argument had a major intellectual consequence. (Here I speak of the history of ancient science and, speaking as a specialist, I allow myself to drop the 'might' peppered through the preceding paragraph.) A crucial feature of élite, literate Greek mathematics (by which I mean the kind of mathematics for which we have evidence in the literary tradition) is its marginalization of the numerical. This is extremely remarkable for a field that, in all other cultures, is mostly organized around numbers. And yet, as noted by Fowler, actual numbers are practically never mentioned in Euclid's *Elements*, a work that *contains three fundamental books on arithmetic*![98] Numbers are only infrequently mentioned elsewhere in mainstream Greek mathematics, usually when simple ratios between geometrical objects are stated. Greek mathematics is centred on geometry and its qualitative features: Greek angles are not measured (inside Greek geometry proper) by degrees, and Greek lines do not have numerically determinate lengths. Relatively late in Greek science, and under the influence of Babylonian antecedents, Greek astronomy became more tied to actual measurable observations, i.e. numerical data. Even then, Greek astronomy remained essentially geometrical in its conception and goals (essentially differing, in this respect, from its Babylonian antecedent). Thus Greek science as a whole was much less quantitative than any other comparable scientific culture. I suggest that this may be to a certain extent explained by the very special Greek hiatus between calculation and argument. Greek science was characterized, as suggested above, by the presentation in writing of verbal persuasion. In Greek mathematics, this verbal persuasion further relied, in an essential way, on a visual cognitive tool: the diagram. The diagram, based on the eye's perception of lines drawn on a surface, easily made the transition to papyrus rolls. Counters, based on manipulability in space, were left out, together with the entire Greek world of calculation. Dotted representations of numbers are found only in late, 'Pythagorean' discussions about mathematics, whereas the mathematical tradition itself ignores them completely. Whenever Greek mathematicians produce the typical Greek text — a mathematical argument with a proof — actual numbers are unimportant, no mention of them is made in the text or in the symbolism, and the argument relies on a combination of natural Greek language and a diagram.[99] This of course should be compared to the development of modern, arithmetized mathematics with its many scientific and practical applications, where the argument is fundamentally tied to numerical representations and to symbols whose function is to relate numbers. The moderns speak of $y = x^2$; the ancients drew parabolas. This, possibly, is the essence of the distinction between ancient and modern science.[100]

The above few paragraphs were, of course, very speculative in nature. The two modern explosions — of capitalism, on the one hand, and of quantitative science, on the other hand — have many forms and causes. I do not suggest that we should ascribe everything to the history of numeracy. To repeat the introduction, this article makes three, more modest claims: that the history of numeracy is worth doing; that it is part of cognitive history; and that Greek numeracy is permeated by counter culture. And it has a single, modest goal: to clarify those claims. I leave it to the reader to judge whether this goal is now fulfilled.

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REFERENCES

1. While the methodology delineated here is, I hope, sufficiently original to justify a programmatic article, it is, of course, not without its antecedents. The best treatment of the history of numeracy, in the modern context, is T. Porter, *Trust in numbers* (Princeton, 1995). Most directly related to this article, I mention P. Damerow and W. Lefevre, *Rechenstein, Experiment, Sprache* (Stuttgart, 1981), an extraordinary study in the history of numerical and empirical practices. While I do not agree with Lefevre's interpretation of Greek counter culture (unfortunately, it is based on some very outdated literature), I found it of great value in developing my own thinking on the subject.
2. See especially J. Goody, *The domestication of the savage mind* (Cambridge, 1977), *The logic of writing and the organization of society* (Cambridge, 1986), and *The interface between the written and the oral* (Cambridge, 1987).
3. There is of course a vast field of studies in reading and writing outside the ancient world, often with interesting consequences for the history of science. One may mention — to give one important example — B. Stock's *The implications of literacy: Written language and models of interpretation in the eleventh and twelfth centuries* (Princeton, 1983).
4. The references are P. C. Cohen, *A calculating people* (Chicago, 1982), a study of elementary mathematical education in early nineteenth-century United States, and J. Brewer and R. Porter (eds), *Consumption and the world of goods* (London, 1994), a study of consumerism as an historical phenomenon.
5. V. Chinapah, *Handbook on monitoring learning achievement* (Paris, 1997).
6. J. Goody, *The interface between the written and the oral* (Cambridge, 1987).
7. In the study of ancient literacy, an important study of this nature is R. Thomas, *Literacy and orality in ancient Greece* (Cambridge, 1992).
8. See e.g. A. W. Crosby, *The measure of reality* (Cambridge, 1997), chap. 6.
9. M. and R. Ascher, *Code of the Quipu* (Ann Arbor, 1981).
10. D. Schmandt-Besserat, *Before writing* (Austin, 1992).
11. See e.g. J. Napier, *Hands* (Princeton, 1980).
12. See S. Dehaene, *Numerical cognition* (Cambridge, Mass., 1993), 12ff.
13. J. M. Pullan, *The history of the abacus* (New York, 1968).
14. A. Schärlig, *Compter avec des cailloux* (Lausanne, 2001).
15. T. L. Heath, *A history of Greek mathematics* (Oxford, 1921), 46–52.
16. I put aside the questions of other methods of calculation. Mental calculation was common, as is clear e.g. from the many papyri containing arithmetical tables: most probably, these were meant to be memorized by heart (on tables of this kind, see D. Fowler, *The mathematics of Plato's Academy* (Oxford, 1999), 234 ff.). Finger reckoning, a very widespread practice across many cultures, was probably practised in the Greek world, too (see e.g. Aristophanes's Vesp. 656–7, where the point may be that the calculation is so simple one can make it with one's fingers, *no need for counters*). As the preceding example shows, both mental and finger reckoning are confined to

relatively simple calculations. They may have served as aids, to further simplify operations on the abacus, but my guess is that, mental calculating prodigies aside, complicated calculations were *always* done with the abacus.

17. As Pullan, *op. cit.* (ref. 13), insists, the abacus in its ancient form was also used in medieval and early modern Europe; I therefore prefer to call it "western", to distinguish it from the eastern abacus. It also should be noted immediately that the abacus was probably not a Greek invention: I shall return to discuss its probable Near Eastern context in Section 3 below.
18. M. L. Lang, "Herodotus and the abacus", *Hesperia*, xxvi (1957), 271–87.
19. M. L. Lang, "Abaci from the Athenian Agora", *Hesperia*, xxxvii (1968), 241–3.
20. Schärlig, *Compter avec des cailloux* (ref. 14), chap. 3 (with references to many recent publications).
21. H. R. Immerwahr, "Aegina, Aphaia-Tempel, an archaic abacus from the Sanctuary of Aphaia", *Archäologischer Anzeiger*, 1986, 195–214.
22. M. L. Lang, "The abacus and the calendar", *Hesperia*, xxxiii (1964), 146–67, and "The abacus and the calendar II", *Hesperia*, xxxiv (1965), 224–57.
23. Lang mentions some of the ancient references in "Herodotus and the abacus" (ref. 18), 271, and more can be found through Pullan, *op. cit.* (ref. 13), 113–14, and Schärlig, *op. cit.* (ref. 20), chap. 1. One should also add Epicharmus fr. 2, an important piece of evidence from a mid-fifth century comedy; and of course the entire literature for the use of counters, or pebbles, in "Pythagorean arithmetic": we shall return to such evidence in 2.4 below.
24. A caveat should be mentioned: Lang was sometimes criticized in the 1960s, most notably in two publications by W. Kendrick Pritchett, "Gaming tables and IG I2 324", *Hesperia*, xxxiv (1965), 131–47 and "'Five lines' and IG I2 324", *California studies in Classical Antiquity*, i (1968), 187–215, for perhaps moving too quickly to conclude that certain artifacts were abaci and not, for instance, game boards. In this article, I try to argue that the attempt to define a sharp distinction between calculation, and other symbolic practices, is in itself misguided, so that the debate, I believe, is about a non-existent question.
25. Note that some use was also made in the Roman world of a more 'eastern' device, where buttons are moved along grooves. See Pullan, *op. cit.* (ref. 13), 19–20.
26. F. Fellmann, "Römische Rechentafeln aus Bronze", *Antike Welt*, xiv (1983), 36–40. The extent to which Roman numeracy differed from its Greek counterpart is an important question I shall not enter into in this article.
27. The "negative position" rule, whereby values are subtracted, not added, when out of the regular decreasing order, is a modern sophistication of Roman numerals. The relation between the abacus and Latin numerals was apparently first noted by C. M. Taisbak, "Roman numerals and the abacus", *Classica et medievalia*, xxvi (1965), 147–60.
28. T. Crump, *The anthropology of numbers* (Cambridge, 1990).
29. In many languages and in particular in all Indo-European languages, the basis of numeral words is decimal; see e.g. T. Dantzig, *Number, the language of science* (New York, 1930), 12.
30. For the following description of Greek numerical record see e.g. Heath, *op. cit.* (ref. 15), 26–45.
31. G. E. M. de Ste. Croix, "Greek and Roman accounting", in *Studies in the history of accounting*, ed. by A. C. Littleton, and B. S. Yamey (London, 1956), 56–57.
32. Like most numeral symbolisms of this kind, the Greek acrophonic numerals use a simple stroke for the unit.
33. J. M. Keynes, *A treatise on money*, i (London, 1930), reprinted in *The collected writings of John Maynard Keynes*, v (Cambridge, 1971), 3, 10.
34. All this has no bearing on the question of the *origin* of coins. Why coins were invented — their *etymology*, as it were — is a question frequently addressed in the literature, thus creating a

somewhat misguided emphasis at least for the interests of this article. Here I am interested not in the question, What made people begin to make objects we identify as "coins"?, but in the question, What made such objects, eventually, so important and stable a feature of the Greek economy? The two questions are likely to get different answers.

35. A further clarification is necessary. It has been argued, rightly I believe, that coins came to have a central place in the Greek symbolic world (see S. von Reden, *Exchange in ancient Greece* (London, 1995), and especially L. Kurke, *Coins, bodies, games and gold: The politics of meaning in Archaic Greece* (Princeton, 1999)). This article may serve perhaps to support this thesis. Here, however, I study primarily not the symbolic world (which has to do, let us say, with what people *think* when they think) but the cognitive world (which has to do with what people *do* when they think). Regardless of what the Greeks may have thought about and through coins, they first had to be able to operate, thinking with them: it is this operability of coins I discuss here.
36. Fowler, *op. cit.* (ref. 16), 227ff. This strict adherence to unit fractions shapes the entire texture of Greek finance (taxes, shares and interest rates were all fixed with reference to unit fractions), and is thus an example of the role of the forms of numeracy in economic history.
37. This is based on a survey of the examples in G. K. Jenkins, *Ancient Greek coins* (London, 1990). It should be noted that diameters vary rather less than weights: with smaller weights, the tendency is to keep the diameter relatively constant while flattening the disk further and further.
38. See e.g. C. Howgego, *Ancient history from coins* (London, 1995), 1–8, especially the map of early minting on p. 5.
39. Aristophanes, *Vesp.* 787–93.
40. See e.g. Jenkins, *op. cit.* (ref. 37), 47, ill. 107.
41. Walter Scheidel, personal communication.
42. See e.g. Jenkins, *op. cit.* (ref. 37), 3. For Athens, see J. H. Kroll, *The Greek coins* (Princeton, 1993), ii (but note that silver-rich Attica was relatively reluctant to introduce bronze coinage: it was much more important e.g. in Sicily).
43. M. L. Lang and M. Crosby, *Weights, measures and tokens* (Princeton, 1964), Part 2.
44. E. E. Cohen, *Athenian economy and society: A banking perspective* (Princeton, 1992), 69. I suspect one did not really need a separate abacus: the table itself would do equally well. The references to the abacus in the banking context may well be references to the table by another name. (To be even more speculative, this may go some way to explain this strange *word* for bank, "table"!)
45. For Babylonian sexagesimals, see e.g. K. A. Menninger, *Number words and number symbols* (Cambridge, Mass., 1969), 162ff. In general, for the role of mathematics in the Babylonian bureaucracy, see E. Robson, *Mesopotamian mathematics, 2100–1600 B.C.: Technical constants in bureaucracy and education* (Oxford, 1999).
46. Thus, for instance, the unit fractions on the Salamis abacus (IG II^2 2777) are easily interpreted as referring to obol and drachma fractions: see Pullan, *op. cit.* (ref. 13), 23–24.
47. Note incidentally that, especially in the banking context, counters designated specially for abacus calculations were superfluous. One could simply use obols: but this is pure speculation.
48. There is very little research of ancient scales and their operation. B. Kisch, *Scales and weights* (New Haven, 1965), has some useful information. M. Lazzarini, "Le bilance romane del museo nazionale e dell antiquarium comunale di Roma", *Atti della Accademia Nazionale dei Lincei*, 8 ser., iii (1948), 221–54, discusses some remarkably sophisticated Roman balances, based on the law of the balance and therefore on a truly mathematical level of numeracy.
49. See e.g. Lang and Crosby, *op. cit.* (ref. 43), 2–4. I should note that I have eaten in the same Hummus joint now for almost fifteen years, and I can swear that the size of the plates has gradually diminished with time.
50. Lang and Crosby, *op. cit.* (ref. 43), 34–38. In a striking metaphor of women as commodity, the

stones are often transparently mastoid (perhaps the metaphor is a crude sexual elaboration of the operation of grasping the weights). Between the thumb and the finger, one held, simply, counters; other grasps called for metaphorical articulation.

51. Lang and Crosby, *op. cit.* (ref. 43), 2–33.
52. De Ste. Croix, *op. cit.* (ref. 31).
53. M. H. Hansen, *The Athenian democracy in the Age of Demosthenes* (Oxford, 1991), 34. An important study, P. Leveque and P. Vidal-Naquet, *Cleisthenes the Athenian: An essay on the representation of space and time in Greek political thought from the end of the sixth century to the death of Plato* (Atlantic Highlands, N.J., 1996; transl. from a work originally publ. 1964), makes the explicit connection between early Greek reflections upon number and space, and the Cleisthenic revolution. While fully aware of the then recent scepticism of Burkert concerning "Pythagoras", Leveque and Vidal-Naquet still posit a direct relation between Pythagoreanism and Cleisthenism. Scholars today might perhaps prefer to think of Cleisthenes not as influenced not by a specific developed doctrine, but by a more diffuse cultural practice: numerical culture.
54. Compare this to modern committees, whose numbers of members is often left variable.
55. Political procedures may make reference to further numerical relations besides simple majority. Thus, e.g., to prevent frivolous litigation, a plaintiff in the law-courts who won less than one-fifth of the votes could face stiff penalties (once again, note the use of a unit-fraction; typically for the Athenian democracy, a simple decimal): see e.g. Hansen, *op. cit.* (ref. 53), 192.
56. Plato *Rep.* 522e.
57. Polybius IX 12–20.
58. For Spartan voting (of which there wasn't much taking place), see e.g. E. S. Staveley, *Greek and Roman voting and elections* (Ithaca, N.Y., 1972), 73ff.
59. G. E. M. de Ste. Croix, *Class struggle in the ancient Greek world* (London, 1981), 413–14. To this, one should compare the closely related conceptual nexus of the notions of coins, novelty and democracy: see the fundamental study by Kurke, *op. cit.* (ref. 35).
60. This interpretation follows Hansen, *op. cit.* (ref. 53), 147–8. Conflicting interpretations of Athenian show of hands were offered in the past, but Hansen's arguments against strict counts seem to me compelling.
61. According to another interpretation of the sources, 6000 was not the quorum for all the ostraka, but was the minimum number required against the *individual* to be ostracized. That our sources can be ambivalent on such a central question is in itself an interesting evidence for the history of Greek numeracy. See e.g. M. L. Lang, *Ostraka* (Princeton, 1990), 1–2, and references there.
62. Ostraka are by their very nature ready-made objects, coming in all shapes and sizes. Still, the voting ostraka fragments preserved (more than 11,000, sometimes in complete shape, and usually allowing an estimate of the original dimensions: see Lang, *op. cit.* (ref. 61)), are all easily graspable by hand: this was crucial, not so much for their casting as votes as for their later counting.
63. The dimensions of such plates were standardized in a rough way — as required by their manipulation. They were on average about 11cm long, 2cm wide and 2mm thick (J. H. Kroll, *Athenian bronze allotment plates* (Cambridge, Mass., 1972), 22).
64. See A. L. Boegehold, *The lawcourts at Athens* (Princeton, 1995), 230–4, and references there.
65. It should be noted that all such votes were of the yes/no type, with rarely — in some trials — the complication of three or four options. Thus no writing was required. One should probably apply this reasoning in the other direction, too: the centrality of the psephoi mode of voting created a strong bias for setting up votes as simple, yes/no decisions.
66. I do not understand how $5n + 1$ numbers were obtained by the $5n$-based kleroterion. One assumes that a final "extra" was obtained through some cruder form of choice by lot.
67. Boegehold, *The lawcourts at Athens* (ref. 64), 214.

68. A. L. Boegehold, "Toward a study of Athenian voting procedure", *Hesperia*, xxxii (1963), 366–74.
69. For seashells see e.g. Boegehold, *The lawcourts at Athens* (ref. 64), 214. Both they and pebbles are plentiful in Attica.
70. *Ibid.*, 234–6.
71. For late law-court procedure as a whole see e.g. Hansen, *op. cit.* (ref. 53), 202.
72. Aristophanes, *Wasps* 94–96.
73. For seating and pay tokens, see Boegehold, *The lawcourts of Athens* (ref. 64), 67–72 and references there.
74. *Ibid.*, p. viii.
75. In Thucydides III.49.1, where the results of the first vote on Mitylene are described, the expression is, significantly, qualitative: the sides were *anchomaloi*, 'nearly equal'.
76. For this contrast between assembly and law-court, see M. H. Hansen, *The Athenian Ecclesia* (Copenhagen, 1983), 110–113.
77. For Spartan pre-coin, iron currency, see e.g. S. Hodkinson, *Property and wealth in Classical Sparta* (London, 2000), 154–76 (Hodkinson notes that foreign coins — valued for their precious metal content — did circulate in Sparta); for Spartan political institutions, see e.g. A. Andrewes, "The government of Classical Sparta", in E. Badian (ed.), *Ancient society and institutions: Studies presented to Victor Ehrenberg on his 75th birthday* (Oxford, 1966), 1–20.
78. Plato *Hip. Mai.* 285c.
79. See e.g. H. W. Parke, *The oracles of Zeus* (Cambridge, Mass., 1967), 129–63; H. W. Parke and D. E. W. Wormell, *The Delphic Oracle*, ii (Oxford, 1956).
80. Pausanias VII.25.10.
81. For divination by counters in general see e.g. W. R. Halliday, *Greek divination* (London, 1913), 205–17.
82. For a discussion of this traditional picture of Pythagorean mathematics, and for a thorough deconstruction of this picture, see Fowler, *op. cit.* (ref. 16), 356ff.
83. W. Burkert, *Lore and science in ancient Pythagoreanism* (Cambridge, Mass., 1972).
84. See in particular the appendix to Fowler, *op. cit.* (ref. 16).
85. Furthermore, games, as a semiotic system, may serve as vehicles of symbolic expression. This symbolic domain is studied as such by the best modern treatment of Greek games, Kurke, *op. cit.* (ref. 35), chap. 7. As I have noted already in the context of coins, this article looks at cognitive, rather than symbolic questions: and I hasten to add, once again, that one set of questions does not rule out the other.
86. The scholarship on Greek sport (for which see e.g. M. Golden, *Sport and society in ancient Greece* (Cambridge, 1998)) usually does not stop to note that there was so much of it *relative to other kinds of games*; but its physical nature, and features such as the relative lack of quantification in ancient sport, are well understood (see e.g. *ibid.*, 61ff).
87. For this probable lineage see H. J. R. Murray, *A history of board-games other than chess* (Oxford, 1952), chap. 6.
88. For ancient board games in general, see Murray, *op. cit.* (ref. 87), chap. 2.
89. Plato *Grg.* 450cd, *Lgs* 819d–820d.
90. See e.g. D. Hoffmann, *Kultur- und Kunstgeschichte der Spielkarte* (Marburg, 1995), 41ff.
91. W. Sombart, *The quintessence of capitalism* (New York, 1967, transl. of 1915).
92. Schmandt-Besserat, *op. cit.* (ref. 10)
93. O. Oppenheim, "On an operational device in Mesopotamian bureaucracy", *Journal of Near Eastern studies*, xviii (1959), 121–8.
94. P. Michalowski, "Early Mesopotamian communicative systems: Art, literature and writing", in

Investigating artistic environments in the ancient Near East, ed. by A. C. Gunter (Washington, D.C., 1990), 53–70.

95. Murray, *op. cit.* (ref. 87), chap. 2.

96. Note that contextuality rules out technological determinism in the strong sense. An individual tool or practice does not lead, in itself, to any historical consequences. That is an appropriate point, and so I stress: I am not a technological determinist in *any* sense. Technology makes nothing necessary. People have ingenuity, which is precisely the ability to practise *against* their setting. Chess has an essential visual component; and people play blind chess. Some Greeks must have had their thumbs cut off by accident. How did they manage? Very well thank you. And yet the tools and practices available at a given culture lend themselves more easily to certain developments than to others; they thus have their role in shaping history.

97. For qualifications — and doubts — concerning de St. Croix's thesis, see R. H. Macve, "Some glosses on 'Greek and Roman Accounting'", in *Crux: Essays in Greek history presented to G. E. M. de Ste. Croix*, ed. by P. A. Cartledge and F. D. Harvey (London, 1985), 233–64, and D. Rathbone, *Economic rationalism and rural society in third-century A.D. Egypt: The Heroninos Archive and the Appianus Estate* (Cambridge, 1991). While evidence for ancient rationalism is not hard to find, it is also clear that rationalism in third-century A.D. Egypt, and in fifteenth-century A.D. Italy, led to different consequences. This, arguably, has to do with the implementation of such rationality with the aid of specific cognitive techniques.

98. Fowler, *op. cit.* (ref. 16), 222.

99. See R. Netz, *The shaping of deduction in Greek mathematics: A study in cognitive history* (Cambridge 1999), chap. 1.

100. See R. Netz, *From problems to equations: A study in the transformation of early Mediterranean mathematics* (forthcoming).