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Activities and Classroom Demonstrations in Biological Physics: A resource document


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Abstract

We give detailed recipes for a number of classroom demonstrations relevant to biological physics instruction. We developed them mainly for use in 2nd-3rd year undergraduate Physics courses. But you can (and we do) use them for primary school, through high school, up to PhD candidates. You adapt the words for each audience, but a phenomenon is a phenomenon.

We also outline a 1-hour introduction to Python for undergraduate scientific computing.

Finally we document some outcomes from courses taught at U Pennsylvania using these resources and others cited here. This material was presented at a workshop at the AAPT meeting in July 2018.

Keywords

biophysics, physics demonstrations, climate

Disciplines

Biochemistry | Biology | Biophysics | Cellular and Molecular Physiology | Molecular Biology | Physical Sciences and Mathematics | Physics

Activities and Classroom Demonstrations in Biological Physics:

A resource document

Philip C Nelson and William Berner, University of Pennsylvania

AAPT meeting 2018

We give detailed recipes for a number of classroom demonstrations relevant to biological physics instruction. We developed them mainly for use in 2nd-3rd year undergraduate Physics courses. But you can (and we do) use them for primary school, through high school, up to PhD candidates. You adapt the words for each audience, but a phenomenon is a phenomenon.

We also outline a 1-hour introduction to Python for undergraduate scientific computing.

Finally we document some outcomes from courses taught at U Pennsylvania using these resources and others cited here.

Bill's other demo shows are also available via a link at www.physics.upenn.edu/~pcn.

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Chapter 1

Intro

Why are we here?

What is interdisciplinary instruction and why is it important? Everyone bemoans the way students silo their knowledge—but who can blame them, when we make little effort to see what’s in their other science classes and make connections? (As do our colleagues in those other departments.)

Also, many students look at Physics and don’t see any connection to something they could do for a living, unlike say Biology, Chemistry, or Engineering.

So let’s see if we can think like physicists while thinking about some phenomena that physicists don’t usually think about. That is one definition of “interdisciplinary.”

An interdisciplinary course can be useful if a practitioner of one discipline shows students how that discipline’s methods have been useful in solving problems of independent interest in another field.

Chapter 2

Skill set

2.1 Wishlist

What do we really want our undergrads to get, as early as possible?

- *Science needs imagination*: “Reason can answer questions, but imagination has to ask them.” — Ralph W Gerard
- *Imagination must be coupled to discipline*: “Seek simplicity, and distrust it.” — Alfred North Whitehead
- *Science involves modeling*: “We all know that Art is not truth. Art is a lie that makes us realize the truth.” — Pablo Picasso
- *Quantitative models are the most falsifiable*: “A single number has more genuine and permanent value than an expansive library full of hypotheses.” — Robert Mayer, 1814–1878

2.2 Specific skills

What physics-style courses can give to life-science students

- Stress that understanding is tested by falsifiable quantitative prediction. Shake the hypothesis till it yields up a testable prediction. Push it to a region of parameter space different from the observations that motivated it in the first place.
- A little familiarity with general-purpose programming language can lower the barrier to everyday tasks—so that they can really become everyday actions: Graphing a function to understand it; visualizing experimental data; switching from linear to log axes; etc.
- Consciously teach the art of throwing out details. The theory should not be more elaborate than the data can support. Don’t stop with a successful prediction—next seek the simplest model that succeeds equally well, and then decide between those two competing hypotheses.

More specific

- Even after two full years of university physics, students have not yet heard (us) say the words “free energy,” even though it’s far more central to chemistry and biology than “energy.”
- They may have encountered the *words* in P-chem, but P-chem is still taught with thermodynamic foundations appropriate for whole moles or reactants—not single molecules. It also spends quite a lot of time on 1920s-era quantum mechanics.
- In fact, first-year physics is basically a rundown of all the rare, mostly irrelevant phenomena that are truly nondissipative, that is, the *exceptions* to normal behavior (except for cursory treatments of friction and electrical resistance, never drawing attention to their dissipative character). Indeed, the entire undergrad program has this blind spot. Even statistical physics is mostly concerned with equilibrium. No wonder students of Chemistry, Biology, Engineering think we’re out on Planet X!
- For those students who have taken Physics, many were never taught any serious optics. (Some study it independently for the GRE, but this is 18th century material.)
- Even among elite US students, most enter university with no useful background in probability, which plays a central role in molecular biophysics.
- Even among elite US students, about half enter university with no useful background in computer programming. Shockingly, nearly the same can be said of undergrads *graduating* from our Physics major, and hence of students entering our PhD program.
- Even among those students who have taken computer programming, it is rare that they got basic science skills like making informative graphs. Those who didn’t take programming have a deeper gap in algorithmic thinking.
- Most physics degree programs, and even moreso other subjects, neglect nonlinear dynamical systems. The intellectual framework needed to understand bifurcations, toggles, oscillators, etc via phase portraits, linearized analysis, etc thus needs to be picked up on the street somehow.
- Most physics degree programs neglect discussion of sensory psychophysics. We go on and on about the crisis of blackbody radiation around 1900, but never discuss single-photon sensitivity of the human visual receptors, which I believe interests students a lot more. Neuroscience courses do pay some homage, but their students lack the background to understand much.

2.3 Filling gaps

Many resources can fill specific gaps in students’ background, for example, khanacademy.org.

Here is a great set of short videos that are specifically tailored to quantitative biology: <http://quant.bio> (Liao, 2018).

Chapter 3

Computation

If you know Python already, think of this as ideas for how to get a class of students going. General starter reference: (Kinder & Nelson, 2018).

3.1 Python

Say to your students: “You need some skills to be a scientist. If you have those skills already in some other language, bear with us and learn them in Python. It will be easier this time around, and anyway some day you may need to work with somebody who only uses Python. And even if today you have free `MATLAB`/*Mathematica* access, some day you won’t have it any more. Python is free and open-source.” (Get it from anaconda.com.)

Some of the suggested actions below will give error messages. Making deliberate errors is good—that way you’ll recognize the messages when you get them *unintentionally*.

3.1.1 First things

= is an action, but == is a question

Start with some basic things in the Console:

```
%reset          #-- you may need to scroll to see the "are you sure?" dialog
1+1             #-- evaluate and print an expression
x = 2*y         #-- error, "y" is not yet defined
a = 1           #-- check in the Variable Explorer window: variable "a" has appeared
a = a+1        #-- check in Variable Explorer window: "a" has been updated
print a         #-- this is an error...
print(a)       #-- print is a function, so it needs parentheses
b == 2         #-- error because == is a relation, not an assignment, and "b" is undefined
a == 2         #-- asks whether the current value in "a" equals 2
a=1; b=a; a=2; #-- check in Variable Explorer: the command b=a did not set
               #-- up a permanent connection.
```

Use the Help pane to get documentation.

Modules

Python isn't very capable when it first launches. You must give it capabilities by importing modules.

```
round(1.4)
sin(1.4)      #-- fails
numpy.sqrt(2) #-- fails until we import
import numpy as np #-- helpful nickname
sin(1.4)      #-- still fails
numpy.sin(1.4) #-- succeeds
np.sqrt(2)    #-- helpful nickname for numpy
np.pi        #-- no parentheses because not a function
from numpy import tan
tan(3.14/4)   #-- don't need module name with this approach
```

Lists are different from arrays

```
a = [3.14, 2.71, 0.577] #-- similar syntax to MATLAB
a[0]                    #-- unlike MATLAB must use square brackets for index
a[1]                    #-- unlike MATLAB, arrays start at 0
```

```
e = [1]
e == 1                 #-- a list with one entry is different from its entry
```

```
b=[[1,2,3],[4,5,6]] #-- represent a matrix as a list of lists
```

Then how to extract a row:

```
b[1]
b[1][0]
b_array = np.array(b) #-- convert b to a numpy array
b_array[1,0] #-- abbreviation for b_array[1][0]; this syntax not available for lists
b_array[1,:] #-- same as b_array[1]
b_array[1,0:1] #-- just part of a row
```

Now: How to extract a column? The answer is `b_array[:,0]`, but you need to understand why.

Variable names are distinct from their objects

```
f = e                #-- defined earlier
e[0] = 3 ; print(f)  #-- surprise, f changes too
e == f              #-- tells you if they currently have same value
e is f              #-- tells you if they are synonyms for a single object
e_array = np.array(e)
f_array = e_array.copy() #-- copy is a method that creates an independent duplicate
e_array == f_array     #-- the == operator acts on each pair of corresponding elements
e_array is f_array     #-- though the values currently agree, they are not same object
e_array[0] = 3000 ; print(f_array)
```

3.1.2 Graphs and vectorization

Graphs

Next a little graph:

```
import matplotlib.pyplot as plt
x = [1,2]; y = [1,2] #-- these create lists, not NumPy arrays, but plot doesn't care
plt.plot(x, y)      #-- helpful nickname for matplotlib.pyplot
```

Vectorization is almost like `MATLAB`, but:

```
y = x**2            #-- fails because vectorization only works on NumPy arrays
y = np.array(x)**2 #-- that works
np.arange?         #-- ask for help on this function
```

```
vals= np.arange(0,3.14)          #-- bad in an instructive way
plt.plot(vals, np.sin(vals))    #-- sin can act on an array
```

Next then repeat with a better choice of `vals`. First, you'll want to clear out the previous bad graphs:

```
plt.close('all') #-- this is a function, and its argument is a string
```

Plot style

There are two basic kinds of plots you need to make: Experimental data (points) and theoretical fit (curves). Figure out how to override the default, which is to join points with lines.

“Help! I can't switch to my plot window!” On macOS, we found that it matters what “Graphics backend” you choose. This one worked for me:

```
python > preferences > iPython console > graphics > backend > Automatic
```

Features of the backend: zoom. Show how it reports coordinates. Save. Figure options.

Play with axis labels, line styles, etc for a while.

Discuss how to have two different plot windows open (`plt.figure()`).

Discuss how to put two plots on the same axes.

3D plotting

```
# code line3d.py
from mpl_toolkits.mplot3d import Axes3D # import 3D plotting tool
fig = plt.figure() # create a new blank figure window
ax = Axes3D(fig) # create 3D plotting object attached to figure
t = np.linspace(0, 5*np.pi, 501) # define parameter for parametric plot
ax.plot(t*np.cos(t), t*np.sin(t), t) # generate 3D plot
plt.show() # often not needed
```

Try dragging inside the plot window to rotate the viewpoint.

Explicit loops

Much of what you need can be done with vectorized code, but not everything.

```
from numpy.random import random
for i in range(10): #-- that colon is essential
    print(random()) #-- indentation is essential
```

(OK, this particular operation actually could have been done with `print(random(10))`.)

Not only is indentation essential, it must also be consistent. Use either tabs or blank spaces, but don't mix them. If you do, you may get indentation that looks good on your screen but that Python will claim is inconsistent.

Get a dataset

Here are some datasets: <https://physicalmodelingwithpython.blogspot.com/p/data-sets.html>. Try going there and clicking `data_sets.zip`.

Now you must either put the dataset you want where Python is going to look, or else tell Python to look where the dataset is located. For the first option, in Spyder console say

```
%pwd
```

This tells you where Spyder will look. (Mnemonic is “print working directory.”) Move a dataset there, for example `01HIVseries/HIVseries.csv`. Confirm that Spyder can see it with

```
%ls
```

(Mnemonic is “list.”) Open and inspect it with

```
%edit HIVseries.csv
```

If it looks good, try reading it with

```
np.loadtxt('HIVseries.csv') #-- error: we didn't tell delimiter
np.loadtxt('HIVseries.csv', delimiter=",") #-- not an error but didn't assign to a variable
a = np.loadtxt('HIVseries.csv', delimiter=",") #-- OK
```

(Mnemonic is “load a text file.”) Then graph it.

The alternate approach is to leave the file where your browser put it. Find it in your computer's file finder and drag/drop it into the IPython console. *This does not read in the file*. However, it does insert the complete “path” into the IPython command line. Before hitting enter, prepend “`%cd` ” (“change directory”; don't forget the blank space separator). Then remove the filename from the end of the path (point only to the folder). Then hit enter. Proceed as before; for the rest of your session, IPython will look for files in the folder you selected.

Fitting data by hand It's better to do this by hand on a couple of examples, and only then introduce black-box software. See (Kinder & Nelson, 2018, §5.2).

3.1.3 Challenge

It can take some refined judgement to make meaningful plots. This is a very instructive exercise in several ways.

```

"""
Author: pcn Python:3.6 PDFexample.py
Description: estimate the probability density function of simulated data
"""

import numpy as np, matplotlib.pyplot as plt
plt.close('all')
from numpy.random import random as rng #-- define convenient nickname
Nsamples = 1000000
x = rng(Nsamples) #**
mycounts, mybins = np.histogram(x,30)
dx = mybins[1] - mybins[0]
bincenters = mybins[0:-1] + dx/2 # discard the last edge, shift others
plt.plot(bincenters, mycounts, '*r')
plt.plot([mybins[0],mybins[-1]], [1,1])
#plt.ylim([0,1.1])

```

Run the code. The solid line doesn't agree with the stars, so correct the stars in the usual way to make them an estimated probability density function. The resulting graph is still hard to interpret, but uncomment the last line and you'll get something more reasonable.

The coding challenge: Replace the fifth line (#**) by:

```
x = np.tan(np.pi*(rng(Nsamples)-.5))
```

The resulting graph is not very informative, and moreover it's quite different every time you run the code! Do whatever it takes to display a nice, meaningful probability density and compare it to the Cauchy form:

$$\varphi(x) = A/(1 + Bx^2). \quad (3.1)$$

Here A and B are two constants that you are to find.

The math challenge: Why does the probability density take that form?

The physics challenge: Are there any real systems that display long-tail distributions like this one?

3.2 Animation

Physical processes unfold over time. Our minds grasp physical mechanisms largely via narrative. So it is not surprising that some of the most vivid physics demonstrations also play out over time. Simulations of physics that unfold over time are similarly powerful; interactive simulations are better; and simulations created by the student can be best of all. This view is becoming mainstream in introductory courses (Chabay & Sherwood, 2015), but the benefits of animated simulation extend farther than this. For examples in the form of student projects, see (Chen et al., 2018).

Any series of graphics can be bundled into a movie file that can be uploaded to YouTube, viewed in a browser, and so on, without requiring the viewer to install Python. Details are given in (Kinder & Nelson, 2018, §7).

Alternatively, the VPython package is great for animation. It doesn't currently work in the Spyder environment, but it works in the Jupyter Notebook environment. First install it:¹ From your computer's the command line use²

```
$ conda install -c vpython vpython
```

The code `flipper.ipynb` shows an animation of stochastic switching between two stable states that captures an essential aspect of chemical reactions, in a way that no static textbook figure could do.

¹Also, glowscript.org lets students run VPython codes without installing anything.

²You can also do this from the Anaconda Launcher application. Some users report problems installing with Windows. First, it is crucial that the graphics card and browser support WebGL, which is GPU-based. A way to check whether there is a problem is to go to <http://webglreport.com/>, which reports whether WebGL in fact works. A related issue is that the graphics driver might need to be updated even if the graphics card does contain a GPU.

Chapter 4

Demo-based modules: Materials

The material I will present today is stuff I generally use for sophomores and juniors. But the beauty of demos is that you can (and I do) use them for second-grade, through high school, up to PhD candidates. You adapt the words for each audience, but a phenomenon is a phenomenon.

Framework One way to structure each module of a class:

- Weird phenomenon: a direct, unfiltered experience of Nature that either defies common sense, or at least seems to conflict with things taught previously.
- Qualitative naive expectation, then proposed corrected expectation; *or* conceptual tools to be introduced.
- Quantitative estimate.
- Calculation. You don't need to work through the complex details of a literal model of your phenomenon—you can boil the phenomenon down to one key point, and find a mathematical system that illustrates just *that*.
- Discuss relevance to an actual biological system or some other department's courses.
- More details and references.

I won't hit every one of these for every module this morning. You can try filling in the missing ones.

4.1 Fluid mechanics is qualitatively different in the microworld

4.1.1 Demo: mixing/unmixing

Materials Corn syrup. Big beaker (\approx 6 inch wide, at least 4 inch tall). Syringe or turkey baster. Food color. Fat stirrer, e.g. a test tube. Overhead projector or document camera.¹

¹Your phone's builtin camera can also be used for this if you clamp it looking downward and feed to the room projector.

How-to Prepare a few ccs of colored corn syrup by mixing with food color, load in syringe.

Pour at least 4 inches of corn syrup into the beaker. Insert the stirrer, positioned at the edge by the wall of the beaker. Place on overhead projector or have students crowd around.

Use syringe to inject colored syrup well below the surface of the bulk clear syrup, but away from the floor or walls of the beaker and away from the stirrer. A cm-size blob will be clearly visible.

Slowly swirl the stirrer 720 degrees around the edge of the beaker. Using exactly the opposite motions, slowly swirl it back in the opposite direction

Weird experience The colored blob responds instantly to distant motion of the stirrer. It also stops moving instantly when the stirrer stops moving. It seems to get mixed, or at least stretched, but ultimately it reforms its original shape and position, as if it “remembered,” over two full turns of the stirrer.

Qualitative naive expectation, then corrected expectation “Miscible fluids never unmix, at least not without elaborate steps like distillation.” “A Newtonian fluid has no memory, for example, no elastic restoration of any remembered state.”

This intuition is based on macroscopic observations with water, which is in a regime dominated by inertia. Other fluids, even water in other scale regimes, are instead dominated by friction, and the qualitative behaviors are different. In fact, even after stirring to the point where the fluids *appear* mixed, they still are not; kinematic reversibility brings them back to nearly the initial configuration.²

The super-convoluted structure of stirred coffee (turbulent flow) allows molecular diffusion to erase a concentration gradient (of ink). But in this demo, there’s no turbulence, so diffusion is negligible. The equations are therefore practically deterministic, and since they are first-order in forces the solution reverses when the force does.

Estimate The shear force law is

$$f/a = \eta \frac{v}{L}.$$

So the material parameter η (“viscosity”) has units $\mathbb{M}/(\mathbb{L}\mathbb{T})$. A more familiar material parameter of a fluid is mass density, \mathbb{M}/\mathbb{L}^3 . Those two numbers characterize a simple (“Newtonian”) fluid mechanically.

There’s *no* combination of these constants with units of length! Fluid mechanics is scale invariant.

But we *can* form a combination with units of *force*, namely η^2/ρ . That critical value is the demarcation between a low-force regime (dominated by friction) and a high-force regime (dominated by inertia). Any simple fluid will have these regimes.

For air, it’s $\approx 2 \cdot 10^{-10}$ N. For corn syrup, it’s $\approx 3 \cdot 10^{-2}$ N. When stirred gently, the corn syrup is in the friction-dominated regime (“laminar flow”).

For water it’s 10^{-9} N. When we stir our coffee, we’re in the inertially-dominated regime (“turbulent flow”). But when a microorganism swims, it’s in the laminar regime.

²This is not complete reversibility! Viscous friction is dissipative; it irreversibly turns mechanical work into *heat*. What is reversible, if we neglect diffusion, is the *bulk motion* of the fluid.

Calculation Poiseuille flow law is not hard to derive, and verify. See (Nelson, 2014).

Discuss relevance to actual biological system or some other department's courses A swimming fish is in the turbulent regime. But interestingly, a swimming *bacterium* is in the laminar regime. This affects strategies for swimming, seeking food, etc. Even *air* has a laminar regime, e.g. relevant to your lungs' cilia.

More details The classics are (Purcell, 1977; Berg, 2004; Berg, 1993). A textbook treatment is in (Nelson, 2014).

Taylor's "unmixing" demo is at <https://www.youtube.com/watch?v=Phsm0c7Hb8Q>, starting around 03:26.

His two mechanical swimmers start at around 06:50 in the same video.

4.2 Energy is conserved, but in surprising ways

4.2.1 Demo: Osmotic flow

Materials

Concentrated solution of table sugar with food coloring.

Visking dialysis tubing (Fisher scientific), or lambskin condom (Rite-Aid).

"Thistle tube," or transparent turkey baster without bulb (referred to below as "glassware").

Big beaker. Clamp stand. Rubber band.

How-to If using dialysis tubing, soak in water first to soften, cut off 25cm, and tie off one end securely. If using pharmacy apparatus, wash inside and out.

Fill the membrane bag partway with sugar solution, test for leaks. Securely attach open end to the wide end of the glassware with rubber band. Fill the rest of the way with sugar solution till the level rises up to the narrow part of the glassware. Do this over a sink to avoid sticky mess. Wash food color off the outside.



Suspend into the beaker using a clamp stand, which also firmly holds the junction between membrane and glassware. Fill the beaker with water, mark the initial level of colored fluid in the glassware, and wait.

Weird experience The bag contains sugar water with a little food color. The bag is not very stretchy, and the water is incompressible, so some is forced up the vertical tube. I immerse the bag in water. It's in mechanical equilibrium. But actually this bag is slightly permeable to water. So the slight additional pressure inside, from the column of water, should drive water through the membrane, slowly lowering the water level in the most boring demo you have ever seen. . . . Wait: The opposite happens instead, which requires doing work against gravity.

Qualitative naive expectation, then corrected expectation Where did the required gravitational potential energy come from? There is plenty of thermal energy from the water bath etc., but "thermal energy never spontaneously converts to mechanical form."

Why do people say that? Energy is conserved, so why shouldn't it be converted from one form to another?

Thermal energy is *disordered*, and the world's disorder doesn't spontaneously decrease. If some gets transformed to a lower-disorder form, somebody must pay the *information bill*; the transformation must be linked to an *increase* in disorder somewhere else that exceeds the thermal decrease.

In our demo, as the volume of interior fluid increases, each sugar molecule gets to range over a bigger volume—an increase in disorder. Up to a point, this can overbalance the thermal decrease required by energy conservation, and the system literally sucks heat out of the environment and does mechanical work.

The flow must eventually stop, because as the process proceeds,

- The increase of disorder with each extra cubic centimeter of flow goes down, because entropy change involves the *relative* volume change, and the total is getting bigger.
- The amount of energy that must be spent for each extra cc goes up, because the weight of the fluid column is increasing.

Estimate Flow stops when *pressure* at the base of the column matches $k_B T$ times the number of sugar molecules divided by their concentration drop. The units work properly.

At room temperature, $k_B T \approx 4 \text{ pN nm}$ (or you may prefer RT/N_0). It's easy to make a sugar solution with concentration 1 M, that is, $6 \cdot 10^{23}$ molecules per liter (10^{-3} m^3). The product of these numbers is $2.4 \cdot 10^6 \text{ N m}^{-2}$, way bigger than an atmosphere!

Calculation

Discuss relevance to actual biological system or some other department's courses Cell membranes are permeable to water, but *not* to ions, so they can and must use osmotic balance to control their size. That is, they evolved *pumps* to actively control size; otherwise, they'd burst or collapse.

More details You may notice that some green color leaks out of the bag into the surrounding water. (Nelson, 2014; Benedek & Villars, 2000a).

https://en.wikibooks.org/wiki/School_Science/Apparatus_for_demonstrating_osmosis

<https://www.youtube.com/watch?v=RhQ5NpYcMOI>

4.2.2 Demo: Rubber elasticity

Materials

Overhead projector, paperclips.

IR camera, rubber sheet.

How-to

a. Make a chain of paperclips. Stretch it straight across the overhead projector plate. Now pound gently on the projector to simulate thermal motion. The straight chain gets distorted and its end-end distance shortens. If you wanted to maintain it straight in spite of the random buffeting, you'd need to supply a constant *force* on the ends.

b. Stretch out the rubber sheet, wait a moment for it to equilibrate. Watch it with the IR camera and suddenly let it relax.

Weird experience The rubber sheet gets *colder* as it relaxes. You can even feel this with a wide rubber band on your lip or other sensitive area.

Qualitative naive expectation, then corrected expectation As it shrinks, the rubber sheet does mechanical work on my hands (exerts a force parallel to motion). Where did the energy to do this work come from? For a steel spring the answer is easy: reduction of internal potential energy. But the individual polymer molecules in rubber are getting *more bent*, not less. So where does the energy being transferred to my hands as mechanical work come from?

As in the osmotic demo, again there is plenty of thermal energy from the surrounding air, but don't professors say, "Thermal energy never spontaneously converts to mechanical form"?

As in the osmotic demo, again the key is to remember that thermal energy is disordered, and the precise statement is that the *entire world's* disorder doesn't spontaneously decrease. If some thermal energy gets transformed to a lower-disorder form (mechanical work, perhaps lifting a weight), somebody must pay the *information bill*; the transformation must be linked to an *increase* in disorder somewhere else that exceeds the thermal decrease. In the case of rubber elasticity, the extra disorder lies in the wiggly shapes of polymer molecules as they revert to a form with shorter end-end distance.

Estimate

Calculation

Discuss relevance to actual biological system or some other department's courses Organisms (you and I) are mostly water. Everything else is mostly polymers (DNA, protein, polysaccharides, ...)! So it's good to know something about their properties. Some, like cartilage, are explicitly designed to buffer mechanical shocks.

More details (Nelson, 2014).

Chapter 5

Demo-based modules: Dynamics

5.1 The flow or release of energy can generate structure

5.1.1 Demo: Nonlinear traveling mechanical wave

Materials Corrugated plastic from Home Depot, such as you'd use to cover a carport. Cut a 2m strip, clip it to a backing, and use a clamp to hold it tilted.

Also need a fine metal chain as long as the plastic. The photo shows the chain in its initial position, trapped in the upper well of a 2-well potential:



How-to Position the chain as shown. Give it a small, subthreshold tweak in the middle; it returns to its metastable state. Give it a larger tweak; two traveling waves ensue moving in opposite directions from the tweak.

Weird experience The waves always rapidly approach a stereotyped form regardless of the character of the tweak. They also hit a terminal velocity.

Qualitative naive expectation, then corrected expectation “Initial conditions matter in dynamics problems.”

But a system with an attractor can forget everything about its initial state except where it is on that attractor.

“How can a wave travel undiminished forever despite friction?”

Each cm of chain has stored gravitational potential energy, but each by itself is in a stable equilibrium. Each is connected to a neighbor, which can pull it over its barrier, and then the next, and so on in a cascade. The rate of release of gravitational p.e. is proportional to the speed of this traveling front. But the rate of frictional energy dissipation is proportional to the *square* of that speed. Enforcing energy balance thus selects a speed, a bit like terminal velocity of a falling body.

What has this got to do with the chemical wave, or with the population-dynamics metaphor? Well, friction and diffusion are both *dissipative* processes.

Estimate Everybody says “The brain is a computer and the nerves are the wires.” But nerve impulses travel at 10 m/s, not the speed of light. What is really going on in this wave? What sets this velocity?

Above I argued that, in the mechanical system, a balance between dissipation (there it is again!) and the release of stored energy sets the wave speed. The same principle applies for nerve impulses. You just have to identify what’s storing energy (capacitance) and what’s dissipating it (electrical resistance).

Each patch of a nerve axon membrane is an excitable medium—like our mechanical system, like the BZ reactants. It’s got a “rest” state with ion channels closed and electric potential drop around -100 mV. It’s also got a “depolarized” state with ion channels open and electric potential drop closer to zero, or even > 0 . Each state is self-perpetuating, but when a localized patch switches state, the change in electric potential spreads diffusively and can flip over neighboring patches. (An extra mechanism resets the system after wave passage, so it’s ready for the next one.)

Here are some measured values:

- radius of squid “giant” axon $a = 0.5$ mm,
- resting membrane conductance $g_{\text{tot}} \approx 5 \text{ m}^{-2} \Omega^{-1}$,
- membrane capacitance per area $C \approx 1 \mu\text{F cm}^{-2}$, and
- the conductivity of salt water $\kappa \approx 3 \Omega^{-1} \text{ m}^{-1}$.

Dimensional analysis won’t take us the whole way, but still there are very few ways to combine these material parameters and end up with dimensions of velocity:¹

$$\sqrt{a\kappa g}/C \sim \frac{\text{m}^{1/2}\Omega^{-1}\text{m}^{-1/2}\text{m}^{-1}}{\text{F m}^{-2}} \sim \text{m} \frac{\text{A volt}}{\text{volt coul}} \sim \text{m/s}.$$

Substituting numbers gives an estimate of wave velocity in the correct ballpark of 10 m/s, and indeed it is a ratio of “spring constant” (C^{-1}) to “resistive” (g^{-1}, κ^{-1}) type factors.²

We can also build a length scale

$$\lambda_{\text{cable}} \equiv \sqrt{a\kappa/g} \approx 17 \text{ mm}$$

and a time scale

$$\tau_{\text{cable}} \equiv C/g \approx 2 \text{ ms}.$$

These are useful and correct for understanding “passive spread” or “electrotonus.”

¹That is, we need coulombs and kilograms to cancel.

²Dissipation yet again.

Calculation

Discuss relevance to actual biological system or some other department's courses Many other examples exist of wavefronts propagating in excitable medium, e.g. cardiac contraction. Pathological wave patterns can lead to sudden cardiac death.

More details (Nelson, 2018a; Nelson, 2014; Benedek & Villars, 2000b, chapt. 10).

5.1.2 Demo: Traveling chemical wave

Materials I found J. Cohen and A. Winfree's simplification of Zhabotinskii's recipe for this reaction in (Stewart, 1998, p250). It uses reagents you can get from your chemistry department, and it's quite reliable. Be sure to use it in a well-ventilated room, and don't forget that the reagents are poisonous:

Prepare in advance four components:

A. 25g of sodium bromate, 335 ml water to dissolve it, then 10 ml concentrated sulfuric acid. **Use all normal precautions for working with sulfuric acid**, e.g. add it to the water and not the other way around. **Use eye protection.**

B. 10g sodium bromide, add water to 100 ml.

C. 10g malonic acid, add water to 100 ml.

D. "1, 10 phenanthroline ferrous complex" (that is not a typo), a redox indicator (Fisher Scientific, or Fisons, Loughborough, UK). This is not a participant; it just lets us see what the other chemicals are doing.

Pre-measure two sets of the four reagents in the quantities mentioned below and bring these to class with two standard (9 cm) glass petri dishes (in case one gets spilled!) and pipettes to deliver precise amounts (see below).

A small class can crowd around and look. For a bigger class you need a document camera. (An overhead projector can generate unwanted vibrations, add heat, etc.)

How-to Put 6ml of A in a glass beaker, then add 0.5ml of B, then quickly mix in 1ml of C. Leave the brown mixture by an open window to lose bromine, until it is the color of pale straw or colorless (2–3 min if agitated in a flat dish).

Add 1ml of D, mix thoroughly, and pour into a petri dish on a level, white, illuminated background. (An overhead projector works, but the heat can create too much convection, and vibration can be a problem too.)

The solution turns patchy blue, then clears to brownish-red. Foci of blue will appear (you may have to wait nervously up to five minutes—very dramatic). They grow into concentric rings, which slowly expand. If you swish the dish to re-mix, the patterns will reappear. (But try waiting for a well-developed pattern, then grabbing the dish and suddenly twisting it about its center; if you're lucky the resulting swirling motion will convert some of the foci from concentric expanding circles to rotating spiral waves!!)

The effect lasts 20–25 min.

Weird experience When we mix oil and water, they macroscopically separate instantly (milliseconds). Here we have aqueous solutions that clearly mix and stay mixed for over a minute. *Then* they form spatial/temporal structures!



Qualitative naive expectation, then corrected expectation “Well-mixed solutions never spontaneously unmix.”

Imagine a population in the wild. The population is autocatalytic (they breed) but self-limiting (they eventually compete for food). There can be two steady states: “low” (breeding not enough to sustain population) and “high” (population stays at maximum carrying capacity). Now imagine somehow preparing a uniform state with population density everywhere right on the cusp. It will be unstable to formation of high or low patches.

Now consider a boundary between two adjacent territories, one “low” and the other “high,” sharing a line border. Population can drift (“diffuse”) across the border, pulling the self-sustaining “high” state down but also pushing the subthreshold “low” state up. In this way one of the two states may be able to convert the other, moving the border. Even chemical reactions can have similar character.

Note that it’s critical to have a distributed source of energy (the initial chemicals themselves, or the food eaten by the population of animals). In our demo, after half an hour, this source of energy gets exhausted, and then everything *mixes and stays mixed*, as thermodynamics instructs us it must do.

(In the demo, an extra mechanism also resets the system after wave passage, so it’s ready for the next one. But there are also conditions supporting only one-shot passage.)

You may say, “Cool demo, but I’m a physicist, and I would feel dishonest if I didn’t myself understand all 14 of the participating chemical reactions, which I don’t.” Indeed, I don’t even understand how sand dunes form, let alone chemistry, but... the point is that one part of the world can get more ordered as long as some other part is getting less ordered. When sand dunes form, mechanical energy of wind is turning into heat. In the BZ reaction, high-quality chemical bond energy is turning into heat. Along the way, some structure is allowed to form.

[Less exciting, but more amenable to analysis, you can discuss *reverse* osmotic flow: Under a sufficient pressure gradient, the sugar solution volume will go *down*, decreasing the freedom of the sugar molecules and hence their entropy. Somebody needs to supply mechanical energy, which winds up as heat, so there’s no conflict with the Second Law.]

Estimate This process is surprising in part because of its unusual time scale (minutes). Where did that time scale come from? Oil and water unmix in milliseconds. It's got something to do with a reaction rate and with diffusion, and *diffusion is very slow* over macroscopic scales.

Calculation

Discuss relevance to actual biological system or some other department's courses There are various examples of pattern formation, e.g. in embryogenesis (millipede needs to make all those segments; snake needs to make all those ribs; hair follicles need to be placed nonrandomly in skin; leopards have spots...).

More details (Strogatz, 2014; Tyson, 1985; Zhabotinsky, 2007).

https://en.wikipedia.org/wiki/Reaction-diffusion_system

https://en.wikipedia.org/wiki/Belousov-Zhabotinsky_reaction

Morphogenesis in embryos: (Phillips et al., 2012).

Here is a recent discovery: (Gelens et al., 2014).

5.2 Dynamical systems can use feedback to generate useful behavior

5.2.1 Demo: Mechanical toggle

Materials This device from Home Depot ("chain tightener"):



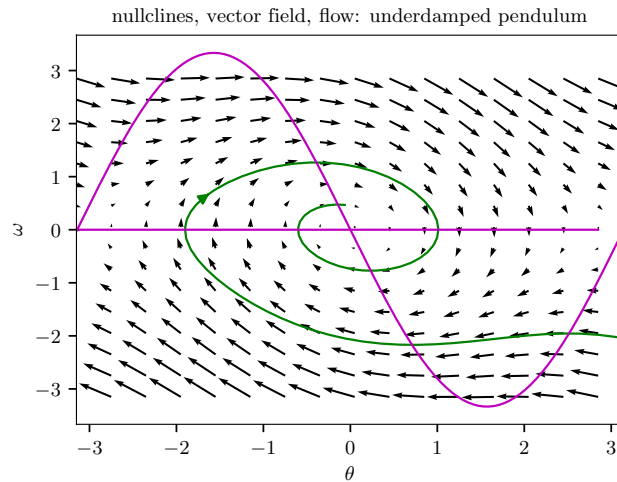
How-to

Weird experience Not so weird. We intuitively understand mechanical bistability.

Conceptual tools This system is a nice introduction to dynamical systems. There is an unstable fixed point flanked by two stable ones, a simple phase portrait.

Estimate

Calculation Compute: Flow of a vector field using `pendulumAAPT.py`. Play with various initial conditions.



Discuss relevance to actual biological system or some other department's courses Cells use negative feedback control to maintain constant internal conditions despite fluctuations in environment, or in internal demand.

Cells use positive feedback control to construct "switchlike" (bistable) dynamics (*lac* and *lambda* switches).

Cells use positive plus negative feedback control to construct oscillatory dynamics (dividing cells in an embryo).

More details (Nelson, 2015).

Chapter 6

Demo-based modules: Optics

6.1 Light resists confinement

6.1.1 Demo: Diffraction through adjustable slit

Materials Laser, adjustable slit. Lab-jack to conveniently position the laser. Lab stand to hold the adjustable slit.

How-to Shine the laser on a distant wall. The beam doesn't spread much. Introduce the slit at its widest setting. The beam still doesn't spread much.

Weird experience Now crank down the slit width. The beam spreads, and also patterns start to form.

Qualitative naive expectation, then corrected expectation Life science students get told that light has a particulate character, but a stream of material particles won't get spread out by confinement, nor will it form these patterns. It won't do to tell them some fairy tale about light being a wave, because the pattern builds up one photon at a time; it is a *probability distribution* for where the next photon is likely to be detected, *not* the amplitude squared of some classical wave.

Estimate Discuss why the expected angular fringe spacing goes up with increasing wavelength; so for dimensional reasons it must go *down* with increasing slit width, i.e. $\Delta\theta \sim \lambda/w$. Multiply by distance to the screen and estimate the expected spacing.

Calculation Work out the 2-slit pattern in detail from Feynman's rule and confirm expectations.

Discuss relevance to actual biological system or some other department's courses The "diffraction limit" was once thought to be a fundamental limitation on optical imaging. Certainly using objects with shorter wavelength (electrons) helps. Reconciling wave and particle aspects is critical to understanding the ongoing revolution in superresolution imaging.

More details (Nelson, 2017; Nelson, 2018b).

6.1.2 Demo: Diffraction of white light by a 1D array sample

Materials 35mm projector, CD, DVD, phonograph record (for comparison), red laser.

How-to Shine directed white light on a CD; get a rainbow. Shine directed monochromatic white light on CD, DVD, phono record: get well-separated spots (unlike 2-slit).

Weird experience What's with those diffraction spots?

Conceptual tools to be introduced Students may have seen the geometric series summation formula, but few will naturally think to apply that to a geometric series whose factor is a complex phase. Yet that very quickly gives a result that predicts sharp, narrow diffraction spots.

Estimate

Calculation

Discuss relevance to actual biological system or some other department's courses Structural color abounds in Nature. Nearly every blue color in a living organism arises in this way (unlike say red, which usually arises via pigments).

6.1.3 Demo: Diffraction of monochromatic light by a 2D or 3D crystalline sample

Materials Red laser (e.g. pointer), Phil's special slide (appendix A).

Other slides are available, e.g. from <http://ice.chem.wisc.edu/Catalog.html>.

Not essential but nice: Blue or UV laser pointer. If you use the UV laser, you'll need a fluorescent target, e.g. large sheet of white paper (a white wall or projection screen will probably *not* be fluorescent).

How-to Shine laser on the slide, observe the pattern.

Weird experience Nice image but... there are no lenses! How do you get a sharp image without lenses? And the resulting pattern doesn't look like what's on the slide. (For one thing, it's far bigger.)

Qualitative naive expectation, then corrected expectation "Objects lit from a distant source cast shadow images."

Instead, in the diffraction regime what we see is the Fourier transform. Qualitatively, the macro- and micro-structures trade places.

Estimate

Calculation

Discuss relevance to actual biological system or some other department's courses This is how the molecular structure of DNA was first obtained, specifically how Rosalind Franklin found its helical structure.¹

More details (Nelson, 2017; Nelson, 2018b). See also Appendix A.

6.2 Biomolecules have characteristic geometries

6.2.1 Demo: Optical rotatory power

Materials Corn syrup and/or sucrose solution. Beaker several inches wide, several inches deep. Polarizing filters big enough to cover the ends of beaker. Overhead projector. Also clear plastic objects (e.g. ruler) and cellophane (e.g. Chinese fortune cookie wrapper).

How-to Put a rigid clear plastic object between crossed polarizers. No light gets through outside the object, but the object itself appears light, and colored.

Place empty glass beaker between crossed polarizers, no effect. Fill it with water, no effect. Dump out water and add 1cm layer of corn syrup or sugar solution and observe. Now add another 1cm, and another. . . .

[Here is another realization: <http://ice.chem.wisc.edu/Catalog.html>.]

Weird experience The polarization gets rotated by an angle proportional to solution depth. But how does that amorphous, indeed liquid, solution know which direction to rotate the polarization?

Qualitative naive expectation, then corrected expectation "An amorphous material (glass or a liquid) cannot do anything at all to the polarization of light because its polarizability tensor gets averaged over all random orientations, becoming effectively isotropic."

Hint: Optical rotatory power breaks inversion symmetry. Inversion-symmetric molecules like water don't display it. But sugar can. No matter how you rotate a chiral molecule, its chirality stays the same.

To go further we must acknowledge *cross-polarizability*. Even a rotationally invariant (isotropic) cross-polarizability can nevertheless break inversion symmetry.

Estimate

Calculation See Chap. 43 from https://repository.upenn.edu/physics_papers/644/ (Nelson, 2018a).

Discuss relevance to actual biological system or some other department's courses This is an essential tool of analytical chemistry. The left-handed ("Z") form of DNA was first documented in this way.

¹Previously Dorothy Crowfoot Hodgkin had solved the structures of cholesterol and penicillin, and others had done myoglobin. Then the stampede started.

More details Some people would *define* a physicist as “scientist obsessed with matters of symmetry,” yet I could not find this phenomenon in any physics textbook, except (naturally) the *Feynman lectures* [§33-5], which however does not do the derivation cited above.

Any p-chem textbook mentions “optical activity,” “optical rotatory dispersion,” and/or “circular dichroism,” but few discuss its symmetry aspects.

This is nice: (Nieves & Pal, 1994).

See also the fascinating story https://en.wikipedia.org/wiki/Louis_Pasteur#Molecular_asymmetry

6.3 We can make quantitative predictions even for unconscious human perception

6.3.1 Demo: $R+G=Y$

Materials

One version: Three 35-mm slide projectors, monochromatic slides

(https://www.pasco.com/prodCatalog/SE/SE-9360_color-filters-set-of-7/), prism.

Alternate version: Theatrical spotlight with sliders for each color (like the Chauvet “Obey3” LED).

How-to

Weird experience Combining pure red and pure green gives a sensation that we do not think of as “reddish green” nor “greenish red”; we experience a unitary sensation called “yellow.” And yet, the components can readily be re-separated using a prism. More disturbing, there is a band in the spectrum that looks exactly the same, yet *cannot* be further separated with a prism.

Qualitative naive expectation, then corrected expectation We think of our senses as telling us “what’s out there.” But really all our brain can know is *what’s coming down the wires*. The brain cannot itself “see” anything. And what’s coming down the wires depends on the transducer hardware. If two kinds of light elicit identical outputs from our photoreceptor cells, then we must say they “match,” *regardless what sophisticated processing is going on*, because we have no other source of information.

We have just three classes of receptors, and two of them have broad, overlapping spectral sensitivity ranges. That’s enough for a qualitative explanation of the phenomenon.

Estimate

Calculation Once the three sensitivity curves have been measured, then predicting the color-matching functions is just a matter of solving three linear equations in three unknowns (Appendix B).

Discuss relevance to actual biological system or some other department’s courses Every intro to psychology pays some lip service to “psychophysics.” Brainbow imaging and spectral karyotyping make use of the insight we got from thinking about human color perception.

More details (Nelson, 2017).

6.4 The color of light is related to energy

6.4.1 Demo: Fluorescence of chlorophyll and other molecules

Materials Need: Black light. Red laser pointer. Bottle of tonic water (supermarket). Acetone (hardware store). Day-glo objects such as fluorescent colored chalk, paper, etc.

Not required but nice: Blue-light LED source. UV laser pointer

(<http://www.arborsci.com/shop-by-topic/light-optics/lasers/laser-pointers>

or <http://www.alpec.com>). Here and below, **use extra precautions with the laser; remember that the UV beam can damage eyes, even though you can hardly see it.**

How-to Prepare chlorophyll solution in advance by soaking any kind of green leaves in acetone overnight. Acetone is nail-polish remover, so work in a well-ventilated place. (I have heard that olive oil also works, but haven't verified that.)

Weird experience Chlorophyll famously looks green under visible light. So does green food coloring. But under UV, chlorophyll gives mysterious blood-red fluorescence, a downconversion (Stokes shift). Normal plant leaves do not do this.

Under visible blue light, chlorophyll scatters some of the blue and downconverts some to that same red.

Tonic water (quinine solution) is also fluorescent. It downconverts UV to pale blue. The UV laser pointer shows that the fluorescence comes off in all directions, despite the narrow incoming beam. In contrast, red laser beam goes straight through unaffected.

Qualitative naive expectation, then corrected expectation Most objects reflect or scatter incoming light into light of the same wavelength. Classical physics explains this by saying the incoming EM wave shakes electrons at its frequency, so they reemit at that same frequency. Even most "colored" objects behave like this, though with differing amount of absorption for each wavelength.

So what's going on with the frequency shift observed in fluorescence? Why is it always toward the red?

We need to get used to thinking of light as bundles carrying definite *energy*. In fluorescence, there is a time delay in the nanosecond range, plenty for the excited molecule to lose some of its excitation energy before emitting a new photon. (There's *dissipation* once again.) See Fig. 6.1 for a physical model of what is going on.

Estimate Pushing the idea forward another step, we also start to understand photoisomerization, the key first step to our vision.

Note that ordinary thermal jostling can in principle achieve the "activation energy" needed to make this conformational change occur spontaneously, without light. But in practice that will never happen at room temperature if the barrier is significantly larger than the typical magnitude of thermal energy at room (or body) temperature, about 4 pN nm.

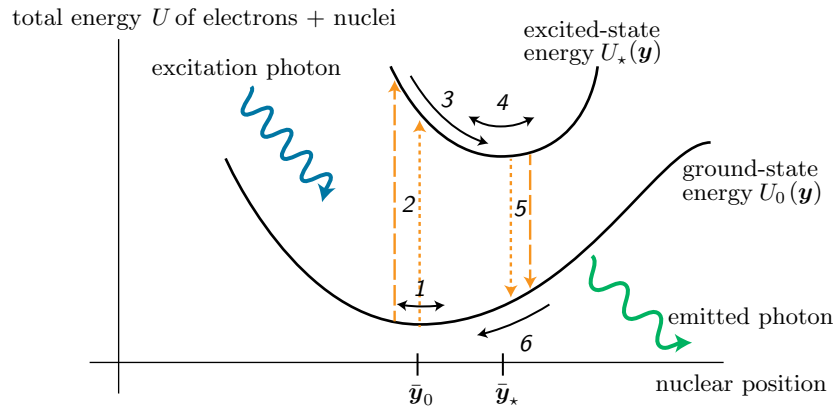


Figure 6.1: **Physical model for fluorescence** from (Nelson, 2017). 1: The positions of the nuclei in a molecule fluctuate near \bar{y}_0 , the minimum of the energy function appropriate for electrons in their ground state. 2: Absorption of a photon promotes the electrons to an excited state. The differing lengths of the *dashed* and *dotted* lines indicate that the required photon energy depends on the momentary value of the nuclear position coordinate y . 3: The nuclei respond by moving toward the minimum of the new, excited-state energy function. 4: Then the nuclei fluctuate about the new minimum \bar{y}_* . 5: Eventually a new photon is emitted (*right*), sending the electrons back to their ground state. The lengths of the *dashed* and *dotted* lines again indicate that there is variation in the precise energy of the emitted photon. 6: The nuclear positions then readjust back to the neighborhood of \bar{y}_0 . The typical difference in length between the ascending and descending arrows represents the Stokes shift of this fluorescent molecule.

Calculation

Discuss relevance to actual biological system or some other department's courses Once you get the realization you find that basically *nothing* important to biophysics requires the wave/Maxwell approach. And nearly *everything* important is in fact *incomprehensible* in that approach!

Fluorescence microscopy is revolutionizing biology.

Once you understand it, then two-photon excitation and Fluorescence Resonance Excitation Transfer (FRET) are small additional steps.

More details (Nelson, 2017; Nelson, 2018b).

6.5 Planetary climate is determined by feedback

6.5.1 Demo: The infrared activity of CO₂

Instrument

We used the FLIR E60 infrared camera (www.flir.com), which contains an uncooled microbolometer array with spectral sensitivity in the range 7.5–14 μm . The camera has video output that can be projected in real time for audience viewing; it can also export still images.² Other cameras may

²We were not able to observe the effect with the much less expensive FLIR One camera.



Figure 6.2: Setup. After irradiation from the two heat lamps (*foreground*), the two cans were viewed by an infrared camera placed on the far side (near the chalkboard). *Not shown*: For the trial shown in Fig. 6.3, each cylindrical can was wrapped in a blanket of foam insulation (except for its ends).

work as long as they have similar sensitivity and spectral sensitivity range, and allow manually fixing a narrow false-color scale. A tripod is helpful.

Materials See Fig. 6.2.

- Four identical coffee cans with both ends removed. Ours were 11.5 cm diameter steel. Optional: we joined two pairs of such cans to get cylindrical chambers of length 23 cm.
- Sheer plastic food wrap (“Glad Wrap/Handiwrap/Saran Wrap”).
- Fire extinguisher or another source of dry CO₂.³
- Two identical sources of infrared radiation, for example, 1000 W ceramic heating elements.
- Apparatus to hold the cans in place (wooden V-blocks, lab jacks, etc).

How-to

1. Cover tube one with food wrap at both ends, secure with rubber bands, and set aside. Cover tube two with food wrap on just one end and secure with rubber band. Prepare a square of food wrap for the last window but do not attach it yet.
2. Set up the tubes with their cylinder axes horizontal (Fig. 6.2), with nothing behind them for at least a meter, and something uniform beyond that (typically a wall of uniform temperature). This horizontal arrangement reduces convective heat transfer from the sources to the cans.

³A CO₂ fire extinguisher is convenient, but beware that most home models are the chemical type, which is not what you want. Some alternative sources: (a) Insert dry ice pellets into a plastic bag in advance, close it until they have sublimated, then pour the gas into the can. (b) Obtain a CO₂ tire inflator and cartridge from a bike shop (e.g. <http://ridepdw.com/>). Chemically generating CO₂ (or letting it escape from a bottle of soda) has the disadvantage of producing gas saturated with water vapor.

3. Turn on the camera and focus on the cans. Align the tubes and camera so that the camera looks straight down each tube's axis. Make sure what the camera sees when looking through each tube is the uniform wall. Then override the automatic temperature scale and manually adjust it: Raise the low extreme of the scale until the wall turns black (e.g. 22°C), then set the upper end about 2°C higher.
4. Place tube two on the floor with open end up and flood it with dry CO₂. Quickly cover it loosely with the remaining food wrap window and wait for any solid CO₂ snow to sublimate, then seal it. Wait for everything to come to room temperature.
5. Image the cans. The experimenter's own emission will create an artifact when specularly reflected from the tube windows, so practice standing to the side to avoid this. Most rooms contain other warm objects, such as computers, so identify and avoid their reflections. For the same reason, audience members should not be too close. [Optionally, it may also be worthwhile to do the demonstration in a darkened room to minimize potential reflections.] Confirm that, with these precautions, both cans appear the same to the infrared camera.
6. Move the sources into position and irradiate the cans for 2 minutes. Point out to the audience that one can easily feel radiation passing out the far ends of each can—at least some wavelengths go right through.
[Caution: Some cameras can “burn in” an image of the hot filament, which will last for some time after removing the sources. To avoid this, block the camera from directly viewing the sources during irradiation.]
7. [Optional: In some trials, we placed a reflecting metal sheet behind the sources during irradiation. This redirects some more energy back toward the cans, and protects the wall of the room from being heated, if space limitations force it to be close to the sources. Alternatively, heat lamps equipped with their own reflectors can be used.]
8. [Optional: In some trials, including the one shown in Fig. 6.3, we blanketed both cans with foam insulation to obstruct energy loss through the can walls. As expected, this change increased the steady-state temperature difference observed after irradiating the cans, but the effect was apparent even without this step.]
9. Suddenly remove the sources (and reflecting sheet if any) and immediately uncover the camera. Image the tubes and observe that the interior of the tube with CO₂ appears warmer than the can with air for at least a minute. More precisely, the apparatus adds its own emission to any wall emission seen through the back end of the can. Again, be sure to stand to the side to avoid imaging yourself.
10. Controls: While practicing the demonstration, it is worthwhile to exchange the positions of the cans, exchange which can gets which gas, exchange the infrared sources, and so on. We found that in every case the only variable relevant to the observed difference was the presence of CO₂ gas.

Weird experience See Fig. 6.3.

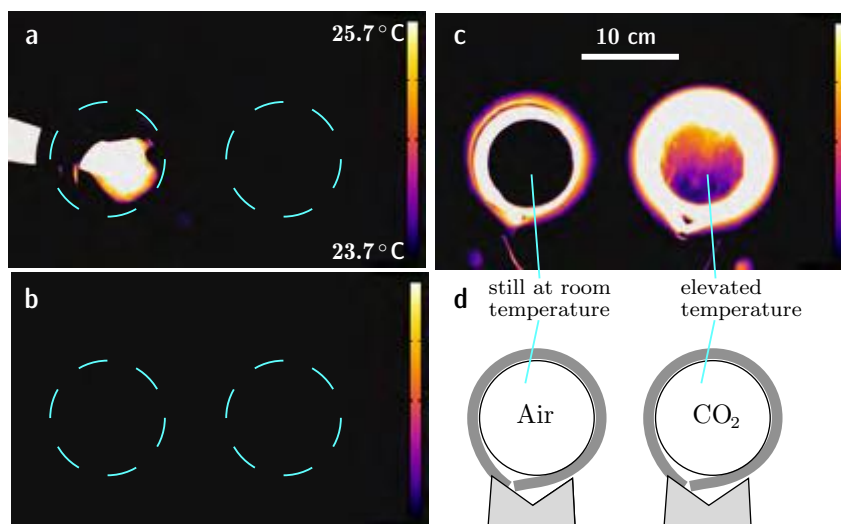


Figure 6.3: (Infrared photographs.) Energy absorption by an infrared-active gas. Two identical cylindrical chambers with transparent ends were viewed in the wavelength band $7.5\text{--}14\ \mu\text{m}$. False color indicates radiance in this band (reds are higher than blues); the scale bar is labeled with approximate inferred temperature in degrees Celsius. The chamber on the left contains dry air. The one on the right contains carbon dioxide. Both are cylinders viewed end-on. a. Chambers before irradiation. A human hand is viewed through the left chamber. *Dashed teal lines* were added to show the locations of the chamber walls. b. The chambers were invisible before irradiation because they matched the ambient room temperature. The backdrop was independently measured to be at about 24°C , which agreed with the camera. c. The chambers were exposed to infrared light for two minutes. After irradiation was stopped, the one containing CO_2 was observed to be slightly warmer for about one minute. That is, the apparent temperature when looking through this chamber was higher than either the backdrop or the other chamber. The bright rings are the steel cans themselves and their insulating blankets. d. Cartoon showing the approximate positions and thicknesses of the foam insulation blankets (*dark gray*). An artifact of the infrared camera makes the warmer right-side insulation appear thicker in (c) than the identical left layer.

Qualitative naive expectation, then corrected expectation Some students are surprised to learn that a gas can be so transparent to some wavelengths (here visible) yet so opaque to others (here IR). But we saw earlier that tonic water is quite transparent to visible, yet strongly absorbs UV, and many other materials are like that.

Estimate Energy transport from a planet's surface outward is notoriously complex, and yet a simple estimate is pretty successful: in the lower part of the atmosphere ("troposphere"), where it is still dense enough to be opaque to IR, temperature depends linearly with altitude, with slope of the graph called the "lapse rate."

For example, dry air (a mixture of gases roughly obeying the ideal gas law, without any component that can condense upon cooling) will have lapse rate $-g/c_p$, where g is the acceleration of gravity at the surface and c_p is the specific heat at constant pressure, per mass. This formula is nearly independent of gas density, because c_p is constant for an ideal gas. The formula just given predicts its value as around 10 K/km.

Similar but more accurate formulas can be written to account for an atmosphere with a condensing component, such as water vapor on Earth. The actual observed lapse rate in the troposphere is about 65 K/(10 km) on Earth, and similar values even on such different planets as Mars and Venus.

Discuss relevance to actual biological system or some other department's courses What is the most important scientific question of all time? Surely the answer is: Climate. So, every academic discipline should be asking, *What can we offer students that is relevant to this question?*

Climate may seem distant from biophysics: The Earth system does not reproduce and hence did not evolve via descent with modification in the presence of natural selection. In that sense, planets are not "alive." But planetary systems are complex and self-regulating, with many interesting feedbacks, so the subject has much in common with the physiology of living organisms. Moreover, our own planet hosts a biosphere that has itself made major modifications to the climate. So whether or not we call it "alive," some of the concepts of biophysics turn out to be relevant.

More details

(Sieg et al., 2019).

P. Nelson, "Planetary Climates": https://repository.upenn.edu/physics_papers/645/

Here is the demo: <https://www.youtube.com/watch?v=0eI9zxZoipA>

Chapter 7

Exams/Assessment

It is safe to say that most Physics exam questions boil down to finding two or three numbers, multiplying/dividing them in the proper way, and emitting a numerical result. This does not always (or ever) engage the sort of skills that we claim it does. Yes, we must retain such questions. But...

Science is largely about careful observation. If I show you a phenomenon, it's fair game for me to ask you *what you saw*, and *how it relates* to the themes of this class. Many students are profoundly uncomfortable with such an open-ended task. They need to get less uncomfortable with it. This is science.

Many students will respond with lots of theory, equations, etc., even when clearly asked *what did you observe*. They need practice before it's time for an exam so that they can do this important task.

Other students are unable to reply concisely to this sort of question; they will fill up a page to prove their erudition, no matter how clearly you tell them to respond in *one sentence*. They need to acquire this important skill of focusing down to just the main point.

With your partner, try constructing a question of the following form, but tailored to each of the experiences we had today:

I did some demonstrations in class. Choose *just xx* of these and write *one sentence* describing the phenomenon that we observed, and *one more sentence* about how that connects to something discussed in class.

- Shine light into a curving "pipe" made of transparent plastic. [sample answer: "Light followed the pipe, even though it was bent and its surfaces were not mirrored." "Total internal reflection as long as every incidence exceeds the critical angle."]
- Diffraction from a triple slit. [sample answer: "Alternating brighter and weaker spots separated by dark bands." "An intermediate to the many-slit pattern, where only the brightest spots survive, and they are very condensed."]

Such open-ended questions may seem hard to grade. So grade them on a coarse scale (0–3 points) and don't make them a huge part of the total. But do include them, just to see if you can make some students think.

Chapter 8

Wrap

Years from now, one out of fifty of your students may remember you vividly as somebody “who showed me how to put it all together.” (*Nobody* will remember you vividly as the person “who got me ready for the MCAT.”) Which one out of fifty is largely a matter of chance—that’s the one who was exactly ready, at that exact moment, to hear about the exact connection you were discussing. What’s *not* a matter of chance is that it cannot happen if you don’t *try* to make connections.

Acknowledgements

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Appendix A

Diffraction slide fabrication notes

Diffraction slide

P. Nelson 10/2011

1 Science

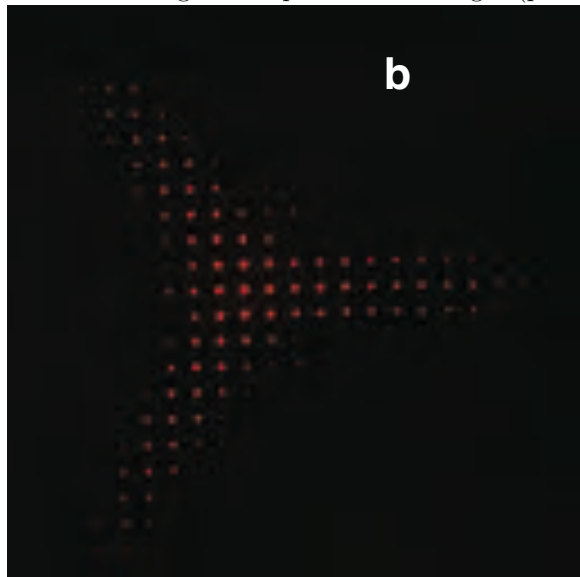
Show a fact about x-ray diffraction (or about Fourier transformation): The fine structure of the scattering object gets encoded in the diffraction pattern as the overall envelope function (the intensities of the spots). Reciprocally, the overall structure of the scattering object (crystal lattice) gets encoded in the fine structure of the diffraction pattern (arrangement of the spots).

2 Fabrication

I prepared `diffractSlideFull.ai` in Adobe Illustrator CS5 (that is, vector art). This file contains 25 panels. Its nominal size is 10 times the actual desired size, because of limitations on the precision of Illustrator's internal representation of lengths. A sampler showing closeups of each of the 25 panels appears at the end of this document.

I sent the file to FineLine Imaging (fineline-imaging.com), indicating the desired final size and specifying 50,800 DPI / $7\mu\text{m}$ feature size and that the image should be rendered positive (white \rightarrow clear, black \rightarrow black). I requested that the image be duplicated as many times as would fit into the minimal order of 117 square inches. FineLine accepts many image file types and converts whatever you send to Gerber format.

Briefly, 24 of the panels contain some equilateral triangles, some hypocycloids (pointier than triangles), or some Y shapes, always arranged in a square lattice. The actual sizes are listed on the last page of these notes. **Big** and **Small** in the titles refer to the size of the objects relative to their spacing. My favorite of these panels was A4 which gave the pattern below right (photo by William Berner):



Panel E1 gives a repeating motif of bar pairs. This one wasn't very successful. I suspect I made the bars too fat, not enough like delta functions, so the bars' own structure function interferes with what I wanted to show, which was the structure factor of the two-bar motif.

MATLAB code `hypocycloid.m` draws these panels, except that the triangles are drawn by `triangleGrid.m`. Save as EPS. Read into Illustrator and save as `.AI`. See comment lines below for hints on Adobe Illustrator postprocessing.

Here is another supplier whom I have not tried: **University Microlitho**
www.microlitho.co.uk university@microlitho.co.uk.

3 Reference

See also Slogoff et al. Photolithographic fabrication of diffraction and interference slit patterns for the undergraduate laboratory. Am. J. Phys. (2004) vol. 72 pp. 1328-1334.

4 Code

```
%% pcn 10/2011 triangleGrid.m
clear all; close all
[x,y]=meshgrid(1:100,1:100);
%axes('Color','w')
%%
h=plot(x,y,'^', 'MarkerEdgeColor','r',...
       'MarkerFaceColor','r',...
       'MarkerSize',1) % size 2 for larger; size 1 and remove the edges and stroke for small
axis equal; axis off
%set(h,'Visible','off')
```



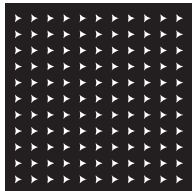
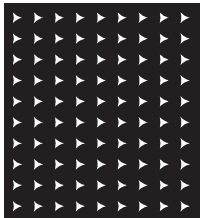
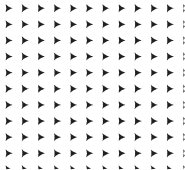
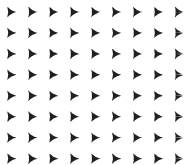
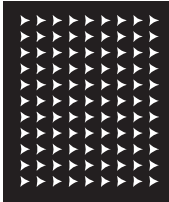
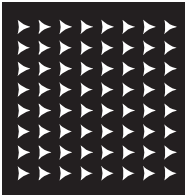
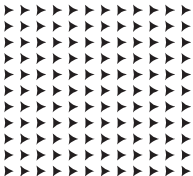
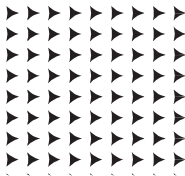
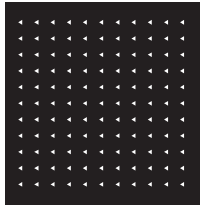
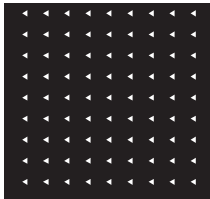
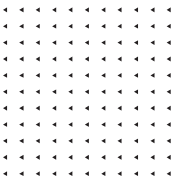
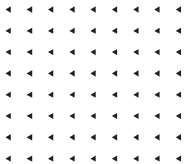
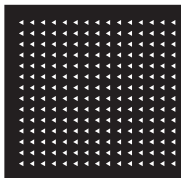
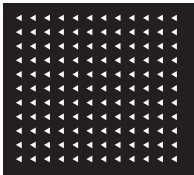
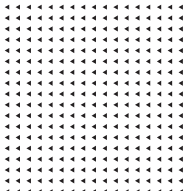
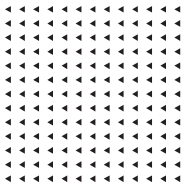


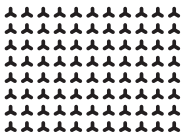
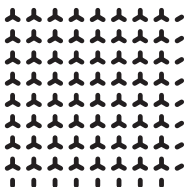
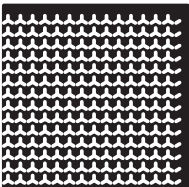
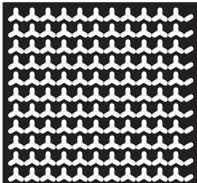
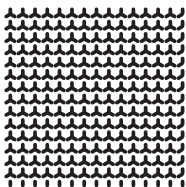
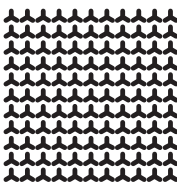

```
%% pcn 10/2011 hypocycloid.m
% Make diffraction slide pictures:
% draw a hypocycloid. draw a 100x100 array of them.
% draw an array of stars.
% draw an array of bar pairs
% postprocessing in Adobe Illustrator: Generally, set fill black and stroke empty. Scale to desired size.
% for Y images, choose rounded ends of lines.
%%
clear all; close all
alpha=.45 %0.4 is approx a triangle; 0.45 is scooped in; 0.5 is very pointy
gamma=2.5 % spacing between the examplars
%% draw a hypocycloid.
theta=[(-pi):(2*pi/21):(pi), -pi]; % make it a closed loop
c=cos(theta);s=sin(theta);
x=(c + alpha*(c.^2-s.^2))/2; y=(s - alpha*(2*c.*s))/2;
%figure(1);plot(theta,sqrt(x.^2+y.^2)); hold on
figure(2); plot(x,y,'k')
axis equal
hold on
%% draw a 100x100 array of them.
figure; hold on;
for i=1:100,for j=1:100
    plot(x+gamma*i,y+gamma*j,'k')
end;end
axis equal; axis off
error('stop')
```



```
%% draw an array of stars.
figure; hold on; axis equal; axis off
s=sin(pi/6);c=cos(pi/6);
for xoff=1:1.5:200, for yoff=1:1.5:200,
    x=[0, 0.1*c, -0.1*c; ...
       0, 0.6*c, -0.6*c]
    y=[0.1, -0.1*s, -0.1*s; ...
       .6, -0.6*s, -0.6*s]
    line(x+xoff,y+yoff,'Color','k')
end;end
hold off
```

```
% now make bar pairs
figure; hold on; axis equal; axis off
line([1:100, (3/8)+(1:100); 1:100, (3/8)+(1:100)], [zeros(1,200); 100*ones(1,200)], 'Color', 'k')
hold off
```

Details of the 25 panels in the diffraction slide. Actual sizes: Panels with names ending in 1 contain 100 objects in 5mm, or 50um spacing. Panels with names ending in 1.3 contain 100 objects in 6.5mm, or 65um spacing. P Nelson 10/2011.

<p>hypoSmallNeg1</p> 	<p>hypoSmallNeg1.3</p> 	<p>hypoSmallPos1</p> 	<p>hypoSmallPos1.3</p> 	<p>hypoBigNeg1 5</p> 
<p>hypoBigNeg1.3</p> 	<p>hypoBigPos1</p> 	<p>hypoBigPos1.3</p> 	<p>triSmallNeg1</p> 	<p>triSmallNeg1.3 4</p> 
<p>triSmallPos1</p> 	<p>triSmallPos1.3</p> 	<p>triBigNeg1</p> 	<p>triBigNeg1.3</p> 	<p>triBigPos1 3</p> 
<p>triBigPos1.3</p> 	<p>YSmallNeg1</p> 	<p>YSmallNeg1.3</p> 	<p>YSmallPos1</p> 	<p>YSmallPos1.3 2</p> 
<p>YBigNeg1</p> 	<p>YBigNeg1.3</p> 	<p>YBigPos1</p> 	<p>YBigPos1.3</p> 	<p>barpairs 1</p> 
A	B	C	D	E

Appendix B

Mechanical analogy for color vision

[Pages from (Nelson, 2017).]

3.8.6 A mechanical analogy for color matching

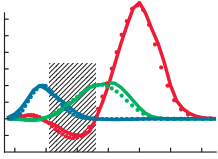


Fig. 3.8 (page 119)

Young's hypothesis appears promising: It explains the $R + G \sim Y$ puzzle, and it can be interpreted in the framework of other things we know about light (Section 3.8.5). But to confirm that it describes our vision, we must show that it correctly predicts the systematics (Section 3.6.3, page 113), as well as the quantitative data obtained from color-matching experiments (Figure 3.8). We can see how to proceed by first making an analogy to a purely mechanical system.

Imagine an air-hockey game. The puck is a disk floating on a thin layer of air. It can move without friction in two directions x and y ; in the third direction, however, it is confined to the plane $z = 0$. We can apply a force \mathbf{F} to the puck and watch it respond. When we do this, we see that it obeys Newton's law of motion:

$$\mathbf{a}_{\perp} = \mathbf{F}_{\perp}/m,$$

where \mathbf{a} is the acceleration vector and the subscript \perp means projection to the xy plane. Let Π denote the projection operation:

$$\mathbf{F}_{\perp} = \Pi(\mathbf{F}) = \begin{bmatrix} F_x \\ F_y \end{bmatrix}. \quad (3.10)$$

The component F_z is irrelevant to the puck's motion.¹⁸ To indicate this, we introduce the equivalence symbol \sim ; the formula $\mathbf{F} \sim \mathbf{F}'$ means that two forces generate the same puck motion, whether or not they are equal. The preceding discussion implies that this statement is the same as saying that $\mathbf{F}_{\perp} = \mathbf{F}'_{\perp}$:

$$\text{Two forces are equivalent } (\mathbf{F} \sim \mathbf{F}') \text{ if they have the same projection, that is, if } \Pi(\mathbf{F}) = \Pi(\mathbf{F}'). \quad (3.11)$$

The relation \sim has some mathematical properties reminiscent of the ones in Section 3.6.3:

- 3'. It is reflexive and transitive.
- 4'. It obeys a Grassmann-type law (page 115).
- 5'–6'. We can choose two fixed "basis" forces $\mathbf{F}_{(1)}$ and $\mathbf{F}_{(2)}$. Then any other "target" force \mathbf{F}^* can be uniquely expressed as equivalent to a linear combination of those two:

$$\mathbf{F}^* \sim \zeta_{(1)}\mathbf{F}_{(1)} + \zeta_{(2)}\mathbf{F}_{(2)}. \quad (3.12)$$

- 7'. There are many allowed choices for the two basis forces. For example, they need not be chosen perpendicular to each other, nor even lie in the xy plane. They must not, however, be parallel to each other, nor may either one be purely vertical.

The origin of all these properties is that the \sim relation amounts to equality of the projections, and projection is a linear function:

$$\Pi(a\mathbf{F} + b\mathbf{F}') = a\Pi(\mathbf{F}) + b\Pi(\mathbf{F}'). \quad (3.13)$$

¹⁸This statements assumes that the force is not too large. Too large a downward force would create friction with the table; too large an upward force would lift the puck off the table.

Your Turn 3E

Confirm that property 4' follows from the fact that Π is linear. That is, show that if $\mathbf{F}_1 \sim \mathbf{F}_2$, then $\mathbf{F}_1 + \mathbf{F}_3 \sim \mathbf{F}_2 + \mathbf{F}_3$.

Ex. Derive properties 5'–7'.

Solution: We wish to show that Equation 3.12 has a unique solution. First, rephrase it as an ordinary vector equation by applying Idea 3.11 and linearity, Equation 3.13:

$$\begin{bmatrix} F_x^* \\ F_y^* \end{bmatrix} = \begin{bmatrix} \zeta_{(1)}F_{(1)x} + \zeta_{(2)}F_{(2)x} \\ \zeta_{(1)}F_{(1)y} + \zeta_{(2)}F_{(2)y} \end{bmatrix}. \quad (3.14)$$

These are two linear equations in the two unknowns $\zeta_{(1)}$ and $\zeta_{(2)}$; every other symbol denotes a known quantity. Solving them in the usual way establishes 5', and is possible as long as the two basis forces have nonzero projections $\mathbf{F}_{(1)\perp}$ and $\mathbf{F}_{(2)\perp}$ that are not parallel to each other. The other properties can be obtained by extending this argument.

To restate the Example in a more general way, we can reduce our problem to a set of two ordinary linear equations once we know $\Pi(\mathbf{F}^*)$ (the left side of Equation 3.14) and the two vectors $\Pi(\mathbf{F}_{(\alpha)})$ (coefficients of $\zeta_{(\alpha)}$ on the right side, for $\alpha = 1, 2$). If you are familiar with matrix notation, you'll recognize Equation 3.14 as the equation

$$\begin{bmatrix} F_x^* \\ F_y^* \end{bmatrix} = \begin{bmatrix} F_{(1)x} & F_{(2)x} \\ F_{(1)y} & F_{(2)y} \end{bmatrix} \begin{bmatrix} \zeta_{(1)} \\ \zeta_{(2)} \end{bmatrix}. \quad (3.15)$$

The condition for a solution to exist is just the statement that the 2×2 matrix in Equation 3.15 should be invertible; moreover, that condition guarantees a unique solution, obtained by multiplying both sides by the inverse matrix.

3.8.7 Connection between the mechanical analogy and color vision

How does the air-puck analogy help us to understand color?

Section 3.8.5 argued that the rate of productive photon absorptions is a linear function of the spectrum (Equation 3.9), analogous to the projection Π in the mechanical analogy (Equation 3.10). In the analogy, Π projects three-dimensional forces to the 2D space of (F_x, F_y) values. In human vision, Equation 3.9 projects the *infinite*-dimensional space of spectra to the *three*-dimensional space of productive absorption rates: the $(\beta_L, \beta_M, \beta_S)$ values. Despite these differences, we can again obtain the color-matching facts in Section 3.6.3 as consequences of the property that the projection operation is linear in \mathcal{I} .

Your Turn 3F

Show that the empirical color-matching rules 3–4 (page 115) follow from our model.

3.8.8 Quantitative comparison to experimentally observed color-matching functions

The quantitative challenge announced in Section 3.8.1 was: Given a target light spectrum \mathcal{I}^* and three basis lights with spectra $\mathcal{I}_{(\alpha)}$, find the amount of each basis light, $\zeta_{(\alpha)}$, needed to match a specified target (or state when no such match is possible).

Before proceeding, we first summarize notation introduced earlier, and extend it somewhat:

Φ_p^*	mean photon arrival rate of a target light (units s^{-1})
$\Phi_{p(\alpha)}$	mean photon arrival rate of basis light α , where $\alpha = 1, 2$, or 3
\mathcal{I}^*	spectral photon arrival rate (“spectrum”) of a target light to be matched (units $\text{s}^{-1}\text{nm}^{-1}$)
λ^*	wavelength in the special case of a target light that is monochromatic (units nm)
$\mathcal{I}_{(\alpha)}$	spectrum of basis light α (units $\text{nm}^{-1}\text{s}^{-1}$)
$\lambda_{(\alpha)}$	wavelength in the special case that basis light α is monochromatic
$\zeta_{(\alpha)}$	mixing coefficient for basis light α (dimensionless)
β_i	mean rate of productive photon absorptions by cones in class $i = L, M$, or S (units s^{-1})
$\mathcal{S}_i(\lambda)$	spectral sensitivity function for cone class i (dimensionless)
β_i^*	projection of the target light spectrum onto cone sensitivity i (units s^{-1})
$B_{(\alpha)i}$	projection of basis light α ’s spectrum onto cone sensitivity i

To address the challenge, we now implement the condition given in Equation 3.6:

$$\mathcal{I}^* \sim \zeta_{(1)}\mathcal{I}_{(1)} + \zeta_{(2)}\mathcal{I}_{(2)} + \zeta_{(3)}\mathcal{I}_{(3)}. \quad [3.6, \text{page 119}]$$

As in the mechanical analogy, first apply the projection (Equation 3.9, page 125) to \mathcal{I}^* , obtaining the three numbers

$$\beta_i^* = \int d\lambda \mathcal{S}_i(\lambda)\mathcal{I}^*(\lambda), \quad i = L, M, \text{ or } S.$$

Similarly, define a 3×3 array of numbers $B_{(\alpha)i}$ by applying the same projection to each of the $\mathcal{I}_{(\alpha)}$. Analogously to Equation 3.14, Equation 3.6 then yields

$$\begin{bmatrix} \beta_L^* \\ \beta_M^* \\ \beta_S^* \end{bmatrix} = \begin{bmatrix} \zeta_{(1)}B_{(1)L} + \zeta_{(2)}B_{(2)L} + \zeta_{(3)}B_{(3)L} \\ \zeta_{(1)}B_{(1)M} + \zeta_{(2)}B_{(2)M} + \zeta_{(3)}B_{(3)M} \\ \zeta_{(1)}B_{(1)S} + \zeta_{(2)}B_{(2)S} + \zeta_{(3)}B_{(3)S} \end{bmatrix}. \quad (3.16)$$

To find the required values of $\zeta_{(\alpha)}$, we solve this set of three linear equations in the three unknowns $\zeta_{(\alpha)}$. All other symbols in Equation 3.16 represent known quantities.

Your Turn 3G

Derive Equation 3.16 from the preceding formulas.

Mathematically, Equation 3.16 always has a unique solution, as long as the three basis lights are suitably generic.¹⁹ However, unlike forces on an air puck, there is no such thing as negative light. Thus, physically all three coefficients $\zeta_{(\alpha)}$ must be *nonnegative* numbers. If a particular target light leads to a solution of Equation 3.16 with any negative values of $\zeta_{(\alpha)}$, then that target cannot be matched with the chosen basis lights.²⁰ For this reason, the color gamut of a computer or television monitor is incomplete; the spectral colors that you see from a prism are more vivid than anything you perceive when looking at a computer screen or printed page.

The experiment of interest to us actually involves a special case of Equation 3.16, which is simpler than the general case: The target lights used to obtain Figure 3.8

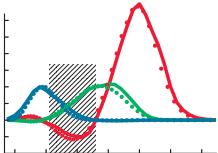


Fig. 3.8 (page 119)

¹⁹If there are fewer than three basis lights, or any basis light is a multiple of another, or one is a combination of the other two, then many target lights cannot be matched. (Mathematically, we say the basis lights must be chosen such that the matrix of $B_{(\alpha)i}$ values is invertible in order to avoid this problem.)

²⁰We already encountered this failure in the experimental data (Figure 3.8, page 119).

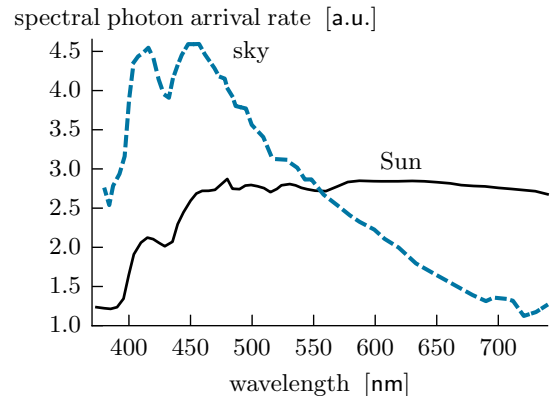


Figure 3.13: [Experimental data.] **Sun and sky.** *Solid curve:* Spectral photon arrival rate of sun-light arriving at Earth’s atmosphere. *Dashed curve:* Spectral photon arrival rate of the blue sky. [Data from Smith, 2005].

were monochromatic. Denote the mean total photon arrival rate of a target light by the symbol Φ_p^* . Then we do not need to integrate over λ when evaluating the left side of Equation 3.16:

$$\beta_i^* = \mathcal{S}_i(\lambda^*)\Phi_p^* \quad \text{for } i = L, M, \text{ or } S. \quad (3.17)$$

Your Turn 3H

In the experiment, the basis lights were also monochromatic. Obtain a formula analogous to Equation 3.17 for the $B_{(\alpha)i}$ in this case.

You’ll finish deriving a prediction for the color-matching experiment in Problem 3.5; that is, by using the empirical sensitivity functions shown in Figure 3.11, together with Equations 3.16–3.17, you’ll predict the values of $\zeta_{(\alpha)}$ as functions of λ^* , and compare them to the experimental data. The result, shown as the solid curves in Figure 3.8, was the main goal of this chapter.

Section 3.8 has shown that the detailed empirical facts about color matching emerge naturally from the physical model of light based on the Light Hypothesis and Electron State Hypothesis, plus the one additional hypothesis that humans determine color by using three classes of photoreceptor cells.

T2 Section 3.8.8’a (page 139) explains why we may use relative sensitivity functions in this derivation in place of the absolute ones. Section 3.8.8’b (page 140) discusses the limited color gamut in more detail.

3.9 WHY THE SKY IS NOT VIOLET

It is often said that a cloudless sky appears blue because Earth’s atmosphere scatters blue light more than it does longer wavelengths. Thus (unlike on the Moon), we see light in directions other than the line of sight to the Sun, but that light has a different spectrum from direct sunlight. Figure 3.13 shows this difference: Light from the sky has more photons at the shortest visible wavelengths than at any longer wavelength.

Faced with that observation, however, we may wonder: If we mostly get violet photons, then why doesn’t the sky appear *violet*? To answer that question, return

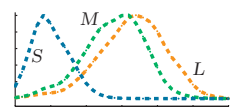


Fig. 3.11 (page 123)

Appendix C








Outcomes

You may need some ammunition to convince colleagues that a course on biological physics is worthwhile. Here are some results from a survey I gave to several years' worth of alumni of my classes.

Initial Report





Last Modified: 11/10/2014

1. I took the course in

#	Answer	Bar	Response	%
1	2013		20	26%
2	2012		10	13%
3	2011		11	14%
4	2010		10	13%
5	2009		12	16%
6	2008		6	8%
7	2007		8	10%
	Total		77	





Statistic	Value
Min Value	1
Max Value	7
Mean	3.44
Variance	4.14
Standard Deviation	2.04
Total Responses	77

2. My level of computer-math experience prior to taking this course was

#	Answer	Bar	Response	%
1	1 = No prior experience		28	36%
2	2		33	42%
3	3		13	17%
4	4 = Extensive prior experience		4	5%
Total			78	





Statistic	Value
Min Value	1
Max Value	4
Mean	1.91
Variance	0.73
Standard Deviation	0.86
Total Responses	78

3. My level of computer-math facility after finishing this course was

#	Answer	Bar	Response	%
1	1 = Inadequate for needs I encountered later		1	1%
2	Click to write Choice 2		8	10%
3	Click to write Choice 3		31	40%
4	4 = Adequate for needs I encountered later		38	49%
Total			78	





Statistic	Value
Min Value	1
Max Value	4
Mean	3.36
Variance	0.52
Standard Deviation	0.72
Total Responses	78

4. Completing this course benefited my work in later courses

#	Answer	Bar	Response	%
1	1 = Not really		6	8%
2	Click to write Choice 2		6	8%
3	Click to write Choice 3		23	30%
4	4 = Significantly		42	55%
	Total		77	





Statistic	Value
Min Value	1
Max Value	4
Mean	3.31
Variance	0.85
Standard Deviation	0.92
Total Responses	77

5. Completing this course benefited my work in later research

#	Answer	Bar	Response	%
1	1 = Not really		7	9%
2	Click to write Choice 2		7	9%
3	Click to write Choice 3		31	40%
4	4 = Significantly		32	42%
	Total		77	

Statistic	Value
Min Value	1
Max Value	4
Mean	3.14
Variance	0.86
Standard Deviation	0.93
Total Responses	77

6. Completing this course led me to take more advanced science course(s) that I might not otherwise have considered

#	Answer	Bar	Response	%
1	1 = Not really		16	21%
2	Click to write Choice 2		11	14%
3	Click to write Choice 3		19	24%
4	4 = Really		32	41%
	Total		78	


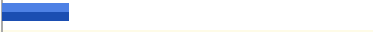


Statistic	Value
Min Value	1
Max Value	4
Mean	2.86
Variance	1.37
Standard Deviation	1.17
Total Responses	78

7. Completing this course contributed to making broader career decisions

#	Answer	Bar	Response	%
1	1 = Not really		11	14%
2	Click to write Choice 2		12	16%
3	Click to write Choice 3		19	25%
4	4 = Really		35	45%
	Total		77	

Statistic	Value
Min Value	1
Max Value	4
Mean	3.01
Variance	1.20
Standard Deviation	1.09
Total Responses	77

8. Completing this course conferred skills that made me more attractive to research labs and/or graduate programs

#	Answer	Bar	Response	%
1	1 = I don't think so		5	6%
2	Click to write Choice 2		14	18%
3	Click to write Choice 3		19	25%
4	4 = I think so		39	51%
	Total		77	

Statistic	Value
Min Value	1
Max Value	4
Mean	3.19
Variance	0.92
Standard Deviation	0.96
Total Responses	77

9. [Optional] Elaborations on these answers, answers to other questions that should have been asked.

Text Response

Just really briefly: There are a lot of classes at Penn that give students the standard experience. Intro Bio, Organic Chem, Calculus, etc. This course (by far one of the best I've ever taken) accomplished an impressive synthesis of high-level biology and physics, acquainting me with the real power of interdisciplinary academia. Courses like this one should be highly encouraged.

I have difficulty assigning a lot of credit specifically to this course for all of the research endeavors I've made and advanced classes I've taken, but it was still undoubtedly a great course that fostered curiosity about biological applications of math and physics.

I would have never pursued the biophysical sciences if I had not taken this class. I am now in a PhD program in biophysics because of how much I loved this class. Prof Nelson was an excellent instructor, and his classes taught me to think in completely different ways about science. I often call the class my most influential class while at Penn.

Will never forget it! Top class I took at penn!

The skills and principles I learned about allowed me to excel at multiple pharmaceutical internships. My expertise is in experimental lab chemistry but the statistics and matlab experience I gained in phys280 allowed me to branch out and do some other experiences in computational chemistry. This class was extremely formative in my science career and I take forward things I learned into my PhD

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I'm now a graduate student in ecology and evolutionary biology at Princeton University. My expertise is in fieldwork and empirical science; this course significantly helped me in three ways: by helping me to better and more easily grasp statistical models of data "from the bottom up" (i.e. based on a fundamental understanding of the probability distributions that are the foundation for the model); by providing me with the computational skills to dabble in simulation and theory to my hearts' content; and by encouraging me to look at the messy biological world conceptually using tractable models.

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Really loved this class. Continue to recommend it to current students. Nelson is a great professor.

Quick dimensional analysis has saved so much time (and so many points) in future classes!

In particular, the skills in building physical models and my understanding of probability/statistics greatly improved my understanding of later courses, as well as proficiency with mathematical computations using programs like MATLAB.

It was difficult to answer some of these questions because my general feelings about the material I learned in biophysics and science in general did not change significantly after the course. I had a high respect for the physical methods in biology before the course and the course only confirmed these feelings. Perhaps if I had taken the course as a freshman it would have had the chance to have an impact on my interests but by the time I took it as a senior my interests had already crystallized in the physical sciences as they relate to biology and therefore there was not much work left to be done. Overall, in my opinion, the single greatest factor in the value of a course is how strongly the instructor is able to make an impress upon the students the importance of the subject matter and to inspire them to work hard to master it. Dr. Nelson succeeded in both of these objectives and therefore I rate the course highly in my mind. My only regret is that I did not have more time my senior year at Penn to devote to mastering the material.

This was absolutely the best class I have ever taken. Phil made me believe in the scientific method and rekindled the purpose and goals of basic research for me.

It was a challenging and engaging course that I am very glad I took -- although I'm not a biology or physics student, I found the material interesting and well-explained.

Most useful class I took at Penn!

To this day, I continue to tell people that PHYS 280 with Phil Nelson was one of my favorite courses in my whole time at Penn (finished a bachelor's and master's)

It was really good to take a computer based science course and it did inspire me to take CIS 120 which I may not have taken otherwise. However, MATLAB is not that common in my field as I assume it is in Biophysics. It might be useful to let physicists with different backgrounds develop with their own preferred language. Although I understand the preference for being able to design problems in a language that you know has a solution that is accessible to Sophomores. In addition, in my year(2011), we spent an incredible amount of time on statistics. I think it would be more useful to give a broader overview of biophysics than drilling into our heads what the mean and variance of the Poisson distribution is.

Still remember the things we went over in this class. The statistics portion was great as was the brief intro to Matlab. Very helpful as a medical student having a quantitative background on some of the physiology we're learning.

My early exposure to Matlab made me more qualified for my research that I am doing currently as well as for other courses I took later on in my undergraduate career.

My valuation of computer-math skills are on the lower end, but that is because I regularly use Matlab on a near daily basis now and Phys 280 was just the first step towards where I am today.

I developed a familiarity and confidence with MATLAB - especially using it to tackle large analysis problems - which still benefits me tremendously today, as a medical student pursuing a master's in clinical research.

Phys 280 was the most useful course I have ever taken. I currently work in predictive marketing analytics, and use the Poisson Process, which was covered in Phys 280 everyday at work. Dr. Nelson framed the course in terms of MLEs and the fundamentals of probabilistic modeling. This framework enabled me to extend my knowledge and derive new statistical models later in my academic career and into my professional career. I also had no knowledge of programming before the course, and now program in R and SQL everyday at work. I learned to program in Phys 280.

While I was drawn to this course because it is about biophysics (which was, and is, my primary scientific interest), the main benefit I took away was the introduction to MATLAB, which has been invaluable to me in my research (I am now a physics Ph.D. student studying biophysics at UCSB).

Having a "mini-introduction" to the course during the few weeks before class starts may be helpful for those that do not have extensive computer-math backgrounds.

Re: 2nd question, I felt more prepared but not entirely prepared for programming applications encountered in later research endeavors, but this is not a fault of the course, the course served as a brief introduction to these skills

This is one of the most rewarding courses I have taken at Penn. This course not only gave me the confidence to take more advanced science classes in the future but also helped inform me on what classes I would find interesting. Since this class emphasized concepts and ideas over facts, my level of retention for this class was a lot higher than for any other class I have taken.

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Phys280 introduced to some extremely important concepts that have become central to my thinking in recent years. No other course I took in the department covers bayesian

reasoning --- an increasingly essential tool.

This was a good course, very unique in its set-up and the accompanying textbook written by the professor (available for free as PDFs, presumably bc it was a work in progress) was very clear and well-written, unlike most actually assigned textbooks which tend to contain weird filler content or be ambiguous, etc. I also enjoyed learning a little Matlab, it was quite fun. I did have some issues applying the knowledge from the class and homeworks to exams. I also remember one of the assignments had us using Matlab to manipulate an image of Dr. Nelson's cat, which is one of the ways the course was particularly charming/endeearing (I don't know if this is an appropriate way to describe the course nor if this is helpful commentary).

I am a math / bio major and this was one of

This is one of the most rewarding courses I have taken at Penn. This course not only gave me the confidence to take more advanced science classes in the future but also helped inform me on what classes I would find interesting. Since this class emphasized concepts and ideas over facts, my level of retention for this class was a lot higher than for any other class I have taken.

This was a fantastic course.

This is one of the most rewarding courses I have taken at Penn. This course not only gave me the confidence to take more advanced science classes in the future but also helped inform me on what classes I would find interesting. Since this class emphasized concepts and ideas over facts, my level of retention for this class was a lot higher than for any other class I have taken.

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Thanks; this was one of the most enjoyable courses I took at Penn and was very influential in directing the type of graduate study I decided to pursue.

Statistic	Value
Total Responses	36