ANALYTICAL MODELS

Ordinary Differential Equations

linear differential equations

$$\frac{d^2u}{dx^2} - x\frac{du}{dx} + u = 0.$$

systems of differential equations

$$R_1(C_1+C_C)\frac{dV_1}{dt} + V_1 = R_1C_C\frac{dV_2}{dt} + V_{DD}$$
$$R_2(C_2+C_C)\frac{dV_2}{dt} + V_2 = R_2C_C\frac{dV_1}{dt}$$

laplace transforms

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt$$
$$H(s) = \frac{1}{(s+\alpha)(s+\beta)}.$$

perturbation expansions

$$\frac{d^2y}{d\tau^2} = -\varepsilon\frac{dy}{d\tau} - 1,$$
$$y(\tau) = y_0(\tau) + \varepsilon y_1(\tau) + \varepsilon^2 y_2(\tau) + O(\varepsilon^3)$$

Difference Equations

discrete time equations

$$d_{n+2} - d_{n+1} - d_n = 0$$

z-transforms

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$
$$H(z) = \frac{k_f^2}{1-(2-k_f(k_f+k_q))z^{-1}+(1-k_fk_q)z^{-2}}$$

Partial Differential Equations

hyperbolic equations / waves

$$\nabla^2\varphi = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$$

parabolic equations / diffusion

$$\nabla^2\varphi = \frac{1}{D}\frac{\partial\varphi}{\partial t}$$

elliptic equations / boundary values

$$\nabla^2\varphi = \rho$$

separation of variables

$$u(\mathbf{r},t) = A(\mathbf{r})T(t).$$
$$A(r,\theta) = R(r)\Theta(\theta),$$

Variational Principles

variational calculus

$$\mathcal{I} = \int_{x_1}^{x_2} f[y(x),\dot{y}(x),x]\, dx$$
$$y(x,\alpha) = y(x) + \alpha\eta(x)$$

euler's equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0$$

constraints and lagrange multipliers

$$\int_{x_1}^{x_2} g[y(x),\dot{y}(x),x]\, dx = C$$
$$h = f + \lambda g$$

Random Systems

random variables

$$\int_{-\infty}^{\infty} f(x)\, p(x)\, dx$$

joint distributions

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, p(x,y)\, dx\, dy$$

bayes' rule

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}.$$

characteristic functions

$$\int_{-\infty}^{\infty} e^{ikx}p(x)\, dx$$

stochastic processes

$$p(x_t|x_{t-\tau_1}, x_{t-\tau_2}, \ldots, x_{t-\tau_N})$$

random number generators

$$x_{n+1} = ax_n + b \ (\mathrm{mod}\ c)$$
$$x_n = x_{n-1} + x_{n-4} + x_{n-6} + x_{n-12}$$

NUMERICAL MODELS

Finite Differences ODEs

euler's method

$$y(x+h) = y(x) + hf(x,y(x))$$

runga-kutta methods

$$k_1 = hf(x,y(x))$$
$$k_2 = hf\left(x+\frac{h}{2},\ y(x)+\frac{k_1}{2}\right)$$
$$k_3 = hf\left(x+\frac{h}{2},\ y(x)+\frac{k_2}{2}\right)$$
$$k_4 = hf(x+h,\ y(x)+k_3)$$
$$y(x+h) = y(x) + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + \mathcal{O}(h^5)$$

predictor-corrector methods

$$y_p(x+h) = y(x) + \frac{h}{24}\{55f[x,y(x)] - 59f[x-h,y(x-h)] + 37f[x-2h,y(x-2h)] - 9f[x-3h,y(x-3h)]\}$$
$$y_c(x+h) = y(x) + \frac{h}{24}\{9f[x+h,y_p(x+h)] + 19f[x,y(x)] - 5f[x-h,y(x-h)] + f[x-2h,y(x-2h)]\} \quad .$$

Finite Differences PDEs

leapfrog method

$$u_j^{n+1} = u_j^{n-1} - \frac{v\Delta t}{\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right)$$

crank-nicholson method

$$\frac{u_j^{n+1}-u_j^n}{\Delta t} = \frac{D}{2(\Delta x)^2}[(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}) + (u_{j+1}^n - 2u_j^n + u_{j-1}^n)]$$

jacobi's method

$$u_{j,k}^{n+1} = u_{j,k}^n + \frac{\Delta t}{(\Delta x)^2}(u_{j+1,k}^n + u_{j-1,k}^n + u_{j,k+1}^n + u_{j,k-1}^n - 4u_{j,k}^n) - \Delta t\rho_{j,k}$$

successive over-relaxation

$$u_{j,k}^{n+1} = (1-\alpha)u_{j,k}^n + \frac{\alpha}{4}(u_{j+1,k}^n + u_{j-1,k}^{n+1} + u_{j,k+1}^n + u_{j,k-1}^{n+1}) - \frac{\alpha(\Delta x)^2}{4}\rho_{j,k}$$

Finite Elements

expansion in basis functions

$$u(\vec{x},t) \approx \sum_i a_i(t)\varphi_i(\vec{x})$$

weighted residuals

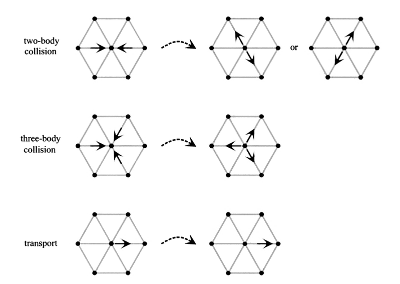
$$R(\vec{x},t) = D[\tilde{u}(\vec{x},t)] - f(\vec{x},t)$$
$$\int R(\vec{x})w_i(\vec{x})\, d\vec{x} = 0$$

rayleigh-ritz variational methods

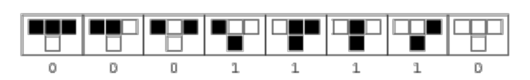
$$\delta\mathcal{I} = \delta\int F[u(\vec{x},t)]\, d\vec{x} = 0$$
$$\frac{\partial\mathcal{I}}{\partial a_i} = 0$$

Cellular Automata & Lattice Gases

lattice gases & fluids



cellular automata & computation



OBSERVATIONAL MODELS

Function Fitting

bayes' rule

$$\max_\varphi\ p(\varphi|d,m) = \max_\varphi \frac{p(d|\varphi,m)\,p(\varphi|m)}{p(d|m)}$$
$$= \max_\varphi \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

maximum likelihood

$$\max_\varphi\ p(\varphi|d) = \max_\varphi\ p(d|\varphi)$$

least squares

$$\min_\varphi \sum_{n=1}^N [y_n - y(x_n,\varphi)]^2$$

gradient descent

$$\chi^2(\vec{a}) = \sum_{n=1}^N \left(\frac{y_n - y(x_n,\vec{a})}{\sigma_n}\right)^2$$
$$\vec{a}_{new} = \vec{a}_{old} - \alpha\nabla\chi^2(\vec{a}_{old})$$

levenberg-marquardt method

$$M_{ii} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial a_i^2}(1+\lambda)$$
$$M_{ij} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial a_i\partial a_j} \quad (i \neq j)$$
$$\delta\vec{a} = -\mathbf{M}^{-1}\cdot\nabla\chi^2$$

Architectures

polynomials

$$y(x) = \sum_{n=0}^N a_n x^n$$

padé approximants

$$y(x) = \frac{\sum_{n=1}^N a_n x^n}{1+\sum_{m=1}^M b_m x^m}$$

splines

$$\vec{x}_i(t) = \vec{c}_{i-3}\varphi_{i-3}(t) + \vec{c}_{i-2}\varphi_{i-2}(t) + \vec{c}_{i-1}\varphi_{i-1}(t) + \vec{c}_i\varphi_i(t) \quad (t_i \leq t < t_{i+1})$$

orthogonal functions

$$y(\vec{x}) = \sum_{i=1}^M a_i f_i(\vec{x})$$
$$\int_{-\infty}^{\infty} f_i(\vec{x})f_j(\vec{x})\, d\vec{x} = \delta_{ij}$$

radial basis functions

$$y = \sum_{i=1}^M a_i f(|\vec{x} - \vec{c}_i|)$$

neural networks

$$X_j = g(\sum_k w_{jk}x_k)$$
$$Y_i = g(\sum_j W_{ij}X_j)$$

Transforms

orthogonal transforms

$$\mathbf{M}^T\cdot\mathbf{M} = \mathbf{I}$$

discrete fourier transform

$$X_f = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} e^{2\pi ifn/N}x_n$$

discrete wavelet transform

$$y_{\text{low}}[n] = \sum_{k=-\infty}^{\infty} x[k]h[2n-k]$$
$$y_{\text{high}}[n] = \sum_{k=-\infty}^{\infty} x[k]g[2n-k]$$

principal component analysis

$$\mathbf{C}_y = \mathbf{M}\cdot\mathbf{C}_x\cdot\mathbf{M}^T$$

Optimization & Search

downhill simplex method

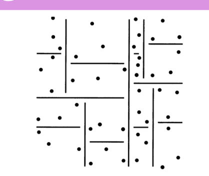
momentum

simulated annealing

genetic algorithms

Clustering & Density Estimation

histograms and k-D trees



fitting densities

$$P(x) = x + \sum_{m=1}^M a_m \sin(m\pi x)$$

kernel density estimation

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{i=1}^n K\left(\frac{x-x_i}{h}\right)$$

mixture models

$$p(\vec{x}) = \sum_{m=1}^M p(\vec{x}|c_m)\ p(c_m)$$

cluster-weighted models

$$p(y,\vec{x}) = \sum_{m=1}^M p(y|\vec{x},c_m)\ p(\vec{x}|c_m)\ p(c_m)$$

Filtering & State Estimation

matched filters

$$f(u) = A\ x^*(t-u)$$

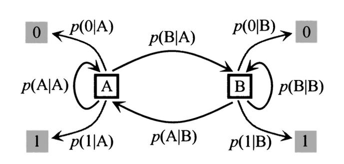
wiener filters

$$F(s) = \frac{C_{x,x+\eta}(s)e^{\alpha s}}{C_{x+\eta,x+\eta}(s)}$$

kalman filters

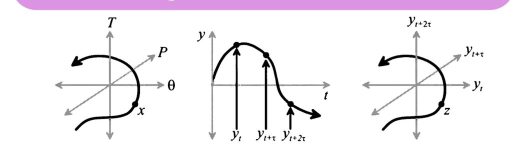
$$\vec{x}_{t|t} = \vec{x}_{t|t-1} + \mathbf{K}_t\cdot(\vec{y}_t - \vec{y}_{t|t-1})$$

hidden markov models



Nonlinear Time Series

state-space reconstruction



dimension measurement

$$D_q \equiv \lim_{l\to 0}\frac{1}{q-1}\frac{\log_2\sum_i p_d(\vec{x}_i)^q}{\log_2 l}$$

lyapunov exponents

$$\lambda = \lim_{t\to\infty}\lim_{\delta\mathbf{Z}_0\to 0}\frac{1}{t}\ln\frac{|\delta\mathbf{Z}(t)|}{|\delta\mathbf{Z}_0|}.$$

entropies

$$H_2(\tau,N) = \sum_{y_t=1}^N\sum_{y_{t-\tau}=1}^N p_2(y_t,y_{t-\tau})\log_2 p_2(y_t,y_{t-\tau})$$